



DEPARTMENT OF MATHEMATICS

UNIT-V LAPLACE TRANSFORM

PARTIAL FRACTION :

Find $L^{-1} \left[\frac{1}{s(s+1)(s+2)} \right]$

Soln: $\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$

$$1 = A(s+1)(s+2) + B(s+2)s + C s(s+1)$$

put $s = -1$

$$1 = A(0) + B(-1)(1) + C(0)$$

$$\Rightarrow B = -1$$

put $s = -2$,

$$1 = A(0) + B(0) + C(-2)(-1)$$

$$1 = 2C \Rightarrow C = \frac{1}{2}$$

put $s = 0$

$$1 = A(1)(2) + B(0) + C(0)$$

$$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$$

$$\frac{1}{s(s+1)(s+2)} = \left[\frac{\frac{1}{2}}{s} - \frac{1}{s+1} + \frac{\frac{1}{2}}{s+2} \right]$$

$$\begin{aligned} L^{-1} \left[\frac{1}{s(s+1)(s+2)} \right] &= L^{-1} \left[\frac{\frac{1}{2}}{s} \right] - L^{-1} \left[\frac{1}{s+1} \right] + L^{-1} \left[\frac{\frac{1}{2}}{s+2} \right] \\ &= \frac{1}{2} L^{-1} \left[\frac{1}{s} \right] - L^{-1} \left[\frac{1}{s+1} \right] + \frac{1}{2} L^{-1} \left[\frac{1}{s+2} \right] \\ &= \frac{1}{2} e^{-t} + \frac{1}{2} e^{-2t} \\ &= \frac{1}{2} [1 - 2e^{-t} + e^{-2t}] \end{aligned}$$



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2) Find $L^{-1} \left[\frac{2s^2 + 4s + 5}{(s+1)(s-3)^2} \right]$

Soln:

$$\left[\frac{2s^2 + 4s + 5}{(s+1)(s-3)^2} \right] = \frac{A}{s+1} + \frac{B}{s-3} + \frac{C}{(s-3)^2}$$

$$2s^2 + 4s + 5 = A(s-3)^2 + B(s-3)(s+1) + C(s+1)$$

put $s=3$,

$$18 + 12 + 5 = A(0) + B(0) + C(4)$$

$$\Rightarrow C = 35/4$$

put $s=-1$,

$$2 - 4 + 5 = A(-4)^2 + B(-4)(0) + C(0)$$

$$\Rightarrow A = 3/16$$

put $s=0$,

$$5 = 9A - 3B + C$$

$$\Rightarrow B = \frac{29}{16}$$

$$\therefore L^{-1} \left[\frac{2s^2 + 4s + 5}{(s+1)(s-3)^2} \right] = \frac{3}{16} L^{-1} \left[\frac{1}{s+1} \right] + \frac{29}{16} L^{-1} \left[\frac{1}{s-3} \right] + \frac{35}{4} L^{-1} \left[\frac{1}{(s-3)^2} \right]$$

$$= \frac{3}{16} e^{-t} + \frac{29}{16} e^{3t} + \frac{35}{4} e^{3t} t$$



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3) Find $L^{-1} \left[\frac{s^2}{(s+1)(s^2+4)} \right]$

Soln: $\frac{s^2}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4}$

$$s^2 = A(s^2+4) + Bs + C(s+1)$$

put $s = -1$

$$\Rightarrow 1 = A(5) + B(-1)(0) + C(0)$$

$$\Rightarrow A = \frac{1}{5}$$

put $s = 0$

$$\Rightarrow 0 = 4A + C \Rightarrow C = -4/5$$

put $s = -4$

$$16 = 20A + 12B - 3C$$

$$\Rightarrow B = \frac{4}{5}$$

$$\therefore L^{-1} \left[\frac{s^2}{(s+1)(s^2+4)} \right] = L^{-1} \left[\frac{1/5}{s+1} + \frac{4/5s - 4/5}{s^2+4} \right]$$

$$= \frac{1}{5} L^{-1} \left[\frac{1}{s+1} \right] + \frac{4}{5} L^{-1} \left[\frac{s}{s^2+4} \right] - \frac{4}{5} L^{-1} \left[\frac{1}{s^2+4} \right]$$

$$= \frac{1}{5} e^{-t} + \frac{4}{5} \cos 2t - \frac{4}{5} \frac{\sin 2t}{2}$$