COMPLEX INTEGRATION

Cauchy Integral Theorem:

If a function f(z) is analytic and its derivative f'(z) is continuous at all point inside and on single closed curve 'c'.

$$\int_C f(z)\,dz = 0$$

Cauchy's Integral formula:

If f(z) is analytic within and on closed curve 'c' and if 'a' is any point within the curve 'c' then

$$f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz$$

(or)

$$\int \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$$

Note:

$$\int_C \frac{f(z)}{(z-a)^2}\,dz = 2\pi i\, f'(a)$$

$$\int_C \frac{f(z)}{(z-a)^3}\,dz = \frac{2\pi i}{2!} f''(a)$$

$\vdots$

$$\int_C \frac{f(z)}{(z-a)^n}\,dz = \frac{2\pi i}{(n-1)!} f^{n-1}(a)$$