3. Evaluate $\int_C \frac{e^z}{(z+1)^2} dz$ where c is the circle $|z-1|=3$

$$\int_C \frac{e^z}{(z+1)^2} dz$$

$$\int_C \frac{f(z)}{(z-a)^2} dz = 2\pi i f'(a)$$ closed curve c

$|z-1| = 3$

$z = a = -1$

$|-1-1| = 3$

$|-2| = 3$

$2 < 3$

$\therefore$ the point lies inside the circle

$f(z) = e^z$

$f'(z) = e^z$

$f'(a) = f'(-1) = e^{-1}$

$$\int_C \frac{e^z}{(z+1)^2} dz = 2\pi i e^{-1}$$

$$= \frac{2\pi i}{e}$$

4. Evaluate $\int_C \frac{z}{(z-1)(z-2)^2} dz$ where c is the circle $|z-2| = \frac{1}{2}$

$$\frac{A}{z-1} + \frac{B}{(z-2)^2}$$

$$\int \frac{z}{(z-1)(z-2)^2} dz$$

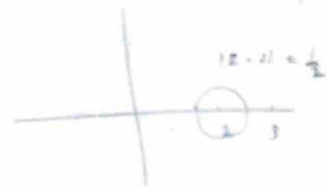
Cauchy's Integral formula is

$$\int_C \frac{f(z)}{(z-a)^2} dz = \frac{2\pi i}{1!} f'(a)$$

Given circle c is $|z-2| = \frac{1}{2}$ ie centre 2 ; radius $= \frac{1}{2}$

$z=1$ lies outside $|z-2| = \frac{1}{2}$

$z=2$ lies inside $|z-2| = \frac{1}{2}$



$$\therefore \int_c \frac{z\,dz}{(z-1)(z-2)^2} = \int_c \frac{\left(\frac{z}{z-1}\right)}{(z-2)^2}\,dz$$

Here $f(z) = \frac{z}{z-1}$, $f'(z) = \frac{(z-1)(1) - z(1)}{(z-1)^2}$

$$= \frac{z-1-z}{(z-1)^2} = \frac{-1}{(z-1)^2}$$

$$\therefore \int_c \frac{f(z)}{(z-2)^2}\,dz = \frac{2\pi i}{1!} f'(2) = 2\pi i \left[\frac{-1}{(2-1)^2}\right]$$

$$= 2\pi i(-1) = -2\pi i$$