



3. Find the residue at the poles of the $f(z) = \frac{z+2}{(z+1)^2(z-2)}$

The poles of $f(z)$ are given by

$$(z+1)^2(z-2) = 0$$

$\Rightarrow z = -1$ is a pole of order 2

$\Rightarrow z = 2$ is a pole of order 1

$$\{ \text{Res } f(z) \}_{z=-1} = \lim_{z \rightarrow -1} \frac{1}{1!} \frac{d}{dz} \left[(z+1)^2 \frac{z+2}{(z+1)^2(z-2)} \right]$$

$$= \lim_{z \rightarrow -1} \frac{d}{dz} \left[\frac{z+2}{z-2} \right]$$

$$= \lim_{z \rightarrow -1} \frac{(z-2)(1) - (z+2)(1)}{(z-2)^2}$$

$$= \lim_{z \rightarrow -1} \frac{z-2-z-2}{(z-2)^2} = \lim_{z \rightarrow -1} \frac{-4}{(z-2)^2}$$

$$= \frac{-4}{(-1-2)^2} = \frac{-4}{(-3)^2} = \frac{-4}{9}$$

$$\{ \text{Res } f(z) \}_{z=2} = \lim_{z \rightarrow 2} (z-2) \frac{z+2}{(z+1)^2(z-2)}$$

$$= \lim_{z \rightarrow 2} \frac{z+2}{(z+1)^2}$$

$$= \frac{2+2}{(2+1)^2} = \frac{4}{9}$$