Invariant or Fixed point:

① Find the fixed point of the function $w = f(z) = \frac{6z-9}{z}$

To find fixed point put $f(z) = z$

$$\Rightarrow \frac{6z-9}{z} = z$$

$$6z - 9 = z^2$$

$$z^2 - 6z + 9 = 0.$$

$$(z-3)(z-3) = 0$$

$z = 3, 3$ are the fixed points

2. $w = \frac{3z-4}{z-1}$

$f(z) = z \Rightarrow \frac{3z-4}{z-1} = z \Rightarrow 3z-4 = z^2-z$

$\Rightarrow z^2 - z - 3z + 4 = 0$

$z^2 - 4z + 4 = 0$

$(z-2)(z-2) = 0$

$\Rightarrow z = 2, 2$

3. $w = \frac{6z+9}{z}$

$f(z) = z \Rightarrow z = \frac{6z+9}{z}$

$\Rightarrow z^2 = 6z + 9$

$z^2 - 6z - 9 = 0$

$(z-3)^2 - 9 - 9 = 0$.

$(z-3)^2 - 18 = 0$

$(z^2-3)^2 = 18$

$z = 3 \pm 3\sqrt{2}$

(or) $z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $a=1$ $b=-6$ $c=-9$

$= \frac{6 \pm \sqrt{36-4(1)(-9)}}{2(1)}$

$= \frac{6 \pm \sqrt{36+36}}{2} = \frac{6 \pm \sqrt{72}}{2}$

$= \frac{6 \pm 6\sqrt{2}}{2} = 3 \pm 3\sqrt{2}$.

4. $w = \frac{2z-5}{z+4}$

$f(z) = z$ $\frac{2z-5}{z+4} = z \Rightarrow 2z - 5 = z^2 + 4z$

$z^2 + 4z - 2z + 5 = 0$

$z^2 + 2z + 5 = 0$.

$(z+1)^2 + 4 = 0$

$(z+1)^2 = -4$

$z + 1 = \pm 2i$

$z = -1 \pm 2i$

5) $w = \frac{z-2}{z+3}$ $\quad f(z) = z$

$f(z) = \frac{z-2}{z+3} \Rightarrow z = \frac{z-2}{z+3} \Rightarrow z^2+3z = z-2$

$z^2+3z-z+2=0$

$z^2+2z+2=0.$

$(z^2+1)^2+2-1=0 \Rightarrow (z^2+1)^2 = -1$

$(z+1)^2 = \pm i$

$z = -1 \pm i$

6. $w = \frac{4z-4}{z}$

$f(z) = z \Rightarrow z = \frac{4z-4}{z}$

$\Rightarrow z^2 = 4z-4$

$z^2-4z+4=0.$

$z = 2, 2$