Unit 3
Complex Differentiation

1. Construct the analytic function $f(z)$ for which the real part is $e^x \cos y$.
2. If f(z) is an analytic function of z prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\mathbf{f(z)}|^2 = 4|\mathbf{f'(z)}|^2$
3. Show that the function $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic and also find its conjugate harmonic.
4. Show that the function $U = \frac{1}{2}\log(x^2+y^2)$ is harmonic and find its harmonic conjugate.
5. Determine the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.
6. Find the image of x=1 under the transformation of $w = \frac{1}{z}$
7. Compute the bilinear transformation which maps the points $z = 0,1,\infty$ into $w = -5,-1,3$ respectively.
8. Find the bilinear transformation which maps ∞,i,0 onto 0,i, ∞
9. If f(z)=u+iv is analytic , find f(z) given that $u + v = e^x(\cos y + \sin y)$