Unit 5
Laplace Transform

1. Determine the Laplace transform of the triangular wave function $f(t)=\begin{cases} t & ,0<t<a \\ 2a-t & ,a<t<2a \end{cases}$

2. Find the Laplace transform of the triangular wave function $f(t)=\begin{cases} t & 0<t<1 \\ 2-t & 1<t<2 \end{cases}$ and $f(t)=f(t+2)$.

3. Using Laplace transforms, solve $y''+6y'+5y=e^{-2t}$ given $y(0)=0, y'(0)=1.$

4. Solve the differential equation using Laplace transform $y''+4y'+4y=e^{-t}$ given that $y(0)=0$ and $y'(0)=0$.

5. 7. Solve by using Laplace transform $y''-3y'+2y=4$ given that $y(0)=2$, $y'(0)=3$.

6. Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$.

7. Using convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$.

8. Find $L^{-1}\left[\frac{1}{s(s-a)}\right]$

$$L^{-1}\left[\frac{1}{s(s-a)}\right]=\int_0^t L^{-1}[F(s)]dt=\int_0^t L^{-1}\left[\frac{1}{s(s-a)}\right]dt$$

$$=\int_0^t e^{at}dt=\left[\frac{e^{at}}{a}\right]_0^t=\frac{1}{a}[e^{at}-1]$$

9. Verify the initial value theorem for 3+4 cos2t

Theorem:

$$\lim_{t\to 0} f(t)=\lim_{s\to\infty} sF(s)$$

$$\lim_{t\to 0}(3+4\cos 2t)=3+4\left[\lim_{t\to 0}\cos 2t\right]=3+4[\cos 0]=7$$

$$\lim_{s\to\infty} sF(s)=\lim_{s\to\infty} sL[3+4\cos 2t]=\lim_{s\to\infty} s[L(3)+4L(\cos 2t)]$$

$$= \lim_{s\to\infty} s[3L(1) + 4L(\cos 2t)] = \lim_{s\to\infty} s\left[3.\frac{1}{s} + 4\frac{s}{s^2+4}\right]$$

$$= \lim_{s\to\infty}\left[3 + 4\frac{s^2}{s^2+4}\right] = \lim_{s\to\infty}\left\{3 + 4\left[\frac{1}{1+\frac{4}{s^2}}\right]\right\}$$

$$= 3 + 4\lim_{s\to\infty}\left[\frac{1}{1+\frac{4}{s^2}}\right] = 3 + 4\left[\frac{1}{1+\frac{4}{\infty}}\right]$$

$$= 3 + 4\left[\frac{1}{1+0}\right] = 7$$