Unit 5
Laplace Transform

1. Verify Initial Value theorem for $f(t)=e^{-t}\sin t$.

Solution:

Initial Value theorem: $\lim_{t\to 0} f(t)=\lim_{s\to\infty} sF(s)$

Now,

$$\lim_{t\to 0} f(t)=\lim_{t\to 0}\left[e^{-t}\sin t\right]=e^{0}\sin 0=1*0=0$$

$$\lim_{s\to\infty} sF(s)=\lim_{s\to\infty} s.L\left[e^{-t}\sin t\right]=\lim_{s\to\infty} s\left[\frac{1}{s^2+1}\right]_{s\to s+1}$$

$$=\lim_{s\to\infty} s\left[\frac{1}{(s+1)^2+1}\right]=\lim_{s\to\infty} s\left[\frac{1}{s^2+2s+2}\right]$$

$$=\lim_{s\to\infty}\frac{s}{s^2\left[1+\frac{2}{s}+\frac{2}{s^2}\right]}=\lim_{s\to\infty}\frac{1}{s\left[1+\frac{2}{s}+\frac{2}{s^2}\right]}=\frac{1}{\infty}=0$$

$\therefore$ **LHS = RHS.**

Hence verified.

2. State Initial value theorem on Laplace Transforms.

If the Laplace transforms of f(t) and f '(t) exist and $L[f(t)]=F(s)$ then $\lim_{t\to 0}[f(t)]=\lim_{s\to\infty}[sF(s)]$

3. State convolution theorem on Laplace Transforms.

If f(t) and g(t) are two functions defined for $t\geq 0$, then

$$L[(f*g)(t)]=L[f(t)].L[g(t)]$$

ie., $L[(f*g)(t)]=F(s).G(s)$ where $L[f(t)]=F(s)$ and $L[g(t)]=G(s)$

4. State the first Shifting theorem on Laplace transforms

If $L[f(t)]=F(s)$ then

$(i) L[e^{at}f(t)]=F(s-a)$

$(ii) L[e^{-at}f(t)]=F(s+a)$

5. Define unit impulse function

The unit impulse function is defined by

$$\delta(t-a)=\begin{cases}\infty, t=a\\ 0, t\neq a\end{cases}$$

such that $\int_{-\infty}^{\infty} \delta(t-a)dt = 1$. It exists only at t = a at which it is infinitely great and is denoted by $\delta(t-a)$.

6. State the Laplace transforms of periodic function with period transforms.

   The Laplace transforms of a periodic function f(t) with period 'p' given by,

$$L[f(t)] = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t)dt$$

7. Find $L^{-1}\left[\frac{1}{s(s-a)}\right]$

$$L^{-1}\left[\frac{1}{s(s-a)}\right] = \int_0^t L^{-1}[F(s)]dt = \int_0^t L^{-1}\left[\frac{1}{s(s-a)}\right]dt$$

$$= \int_0^t e^{at}dt = \left[\frac{e^{at}}{a}\right]_0^t = \frac{1}{a}[e^{at} - 1]$$

8. State the condition for existence of laplace Transform
   (i) f(t) should be continuous or piecewise continuous in the given closed interval [a,b] where a>0.
   (ii) f(t) should be of exponential order.