Unit 2
Ordinary Differential Equations

1. Solve : $\left(D^2 - 2D + 2\right)y = 0$.

   $The\ Auxillary\ Equation\ is\ m^2 - 2m + 2 = 0$

   $m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$

   $The\ Complete\ Solution\ is\ y = e^x \left(A\cos x + B\sin x\right)$

2. Solve : $\left(D^2 - 6D + 13\right)y = 0$.

   $The\ Auxillary\ Equation\ is\ m^2 - 6m + 13 = 0$

   $m = \frac{6 \pm \sqrt{36-52}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$

   $The\ Complete\ Solution\ is\ y = e^{3x} \left(A\cos 2x + B\sin 2x\right)$

3. : Solve $(D^2 + 2)y = 0$

   The Auxillary Equation is $m^2+2=0$

   m=$\pm\sqrt{2i}$

   The CF is given by $Acos\sqrt{2}x + Bcos\sqrt{2}x$

4. Solve : $\left(D^2 + 1\right)y = e^{-x}$.

   $The\ Auxillary\ Equation\ is\ m^2 + 4 = 0$

   $\left(m^2 + 2^2\right) = 0$

   $m = \pm 2i$

   $The\ Complementary\ Function = A\cos 2x + B\sin 2x$

   $P.I = \frac{1}{D^2 + 4} e^{-2x} = \frac{1}{8} e^{-2x}$

   $The\ Complete\ Solution\ is\ y = A\cos 2x + B\sin 2x + \frac{1}{8} e^{-2x}$

5. Transform the equation into a linear equation with constant coefficient

$$(2x-1)^2\frac{d^2y}{dx^2}-4(2x-1)\frac{dy}{dx}+8y=8x$$

6. Find the particular integral of $\left(D^2+1\right)y=\cos(2x-1)$

$$P.I=\frac{1}{\left(D^2+1\right)}\cos(2x-1)=\frac{1}{(-4+1)}\cos(2x-1)=-\frac{1}{3}\cos(2x-1)$$

7. Find the particular integral of $(D+1)^2y=e^{-x}\cos x$.

$$P.I=\frac{1}{(D+1)^2}e^{-x}\cos x=e^{-x}\frac{1}{((D-1)+1)^2}\cos x$$

$$=e^{-x}\frac{1}{D^2}\cos x=e^{-x}\frac{1}{-1}\cos x=-e^{-x}\cos x$$