SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

COIMBATORE – 35

23MAT103 – DIFFERENTIAL EQUATIONS AND TRANSFORMS

UNIT – IV

FOURIER SERIES & FOURIER TRANSFORM

1. Pick out the even function $x^2, \ \sin x, \ 1+x, \ e^x$

   a. x^2 b. $x^2, \sin x$

   c. e^x d. x^2, e^x **Ans : a**

2. The period of tan x is

   a. $\pi/2$ b. π

   c. $3\pi/2$ d. 2π **Ans : b**

3. The period of sin x is

   a. $\pi/2$ b. π

   c. $3\pi/2$ d. 2π **Ans : d**

4. The value of a_0, when the odd function f(x) is expanded in (-π, π) is

   a. 0 b. 2π

   c. $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ d. π **Ans : a**

5. The value of a_0 in the Fourier series of f(x) = x in (0, 2π) is

   a. 0 b. 2π

   c. 4π d. π **Ans : b**

6. The value of b_n in the expansion of x^2 as a Fourier series in (-π, π) is

   a. $2\pi^3/3$ b. $\pi^3/3$

   c. $3\pi^2/2$ d. 0 **Ans : d**

7. The Fourier series expansion of even function contains

   a. Sin terms only b. Cosine terms only

   c. Both d. Neither sine nor cosine **Ans : b**

8. The Fourier series expansion of an odd function contains

   a. Sin terms only b. Cosine terms only

   c. Both d. Neither sine nor cosine **Ans : a**

9. The value of a_n in the Fourier series of f(x) in (0, l) is

   a. $\frac{1}{l}\int_0^l f(x)\cos nx\,dx$ b. $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c. $\frac{2}{l}\int_0^l f(x)\cos nx\,dx$ d. $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ **Ans : d**

10. The value of a_0 in the Fourier series of f(x) in (0, l) is

a. $\frac{2}{l}\int_0^l f(x)\cos nx\,dx$ b. $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c. $\frac{2}{l}\int_0^l f(x)dx$ d. $\frac{2}{l}\int_0^l f(x)\sin nx\,dx$ **Ans : c**

11. The value of a_0 in the Fourier series of f(x) = K, 0 < x < 2π is

a. K b. 2K

c. 3K d. 4K **Ans : c**

12. The value of the constant term in the Fourier series corresponding to $f(x) = x - x^3$ in (-π, π) is

a. $\pi - \pi^3$ b. π

c. π^3 d. 0 **Ans : d**

13. To what the values, the Fourier series corresponding to $f(x) = x^2$ in (0, 2π) converges at x = 0

a. $2\pi^2$ b. π^2

c. $\pi^2/2$ d. $4\pi^2$ **Ans : a**

14. The value of the constant a_0 in the Fourier series of $x\ cosx, -\pi < x < \pi$ is

a. $\pi/3$ b. $2\pi/3$

c. Π d. 0 **Ans : d**

15. The value of the constant a_0 in the Fourier series of $(\pi - x)^2/4,\ 0 < x < 2\pi$ is

a. $2\pi^2/3$ b. $\pi^2/3$

c. $\pi^2/6$ d. π^2 **Ans : c**

16. The Fourier series of $f(x) = (\pi - x)^2/4,\ 0 < x < 2\pi$ converges to ________ at x = 0

a. $2\pi^2/3$ b. $\pi^2/3$

c. $\pi^2/4$ d. π^2 **Ans : c**

17. Fourier series of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ converges to ________ at x = 0

a. $\pi/2$ b. $-\pi/2$

c. 0 d. $\pi/3$ **Ans : c**

18. If f(x) is defined in (-2, 2), then b_n is

a. $\frac{1}{4}\int_{0}^{2} f(x)\ dx$ b. $\frac{1}{2}\int_{0}^{2} f(x)\cos\frac{n\pi x}{2}dx$

c. $\frac{1}{2}\int_{-2}^{2} f(x)\sin\frac{n\pi x}{2}dx$ d. $\frac{1}{2}\int_{-2}^{2} f(x)\sin\frac{nx}{2}dx$ **Ans : c**

19. The value of the a_n in the Fourier expansion of K, on (0, 10) is

a. 10 b. 2K/nπ

c. Π d. 0 **Ans : d**

20. To what value , the Fourier series corresponding to $f(x)=\begin{cases}\pi+x, & -\pi<x<0\\ \pi-x, & 0<x<\pi\end{cases}$ converges at x = 0

a. π□ b. 0

c. 3π d. 2π **Ans : a**

21. To what value, the Fourier series corresponding to $f(x)=\pi-x/2,\ \ 0<x<2\pi$ converges at x = 2π

a. □ π/2 b. 0

c. π/4 d.□ 2π **Ans : b**

22. If a periodic function f(x) is even in *(-l, l)*, then a_0 □is

a. $\frac{1}{l}\int_{0}^{l} f(x)\ dx$ b. $\frac{2}{l}\int_{0}^{l} f(x)\ dx$

c. $\frac{2}{l}\int_{-l}^{l} f(x)\ dx$ d. 0 **Ans : b**

23. The function $f(x)=\frac{1}{1-x}$ is

a. Continuous at x = 1 b. Discontinuous at x = 1

c. Continuous for all x d. Discontinuous at x = 0 **Ans : b**

24. The value of the b_n in the Fourier expansion of $f(x)=|\sin x|$, in (-π, π) is

a. 0 b. 2π

c. π d. 4π **Ans : a**

25. The value of the b_n in the Fourier expansion of $f(x)=x\cos x$, in (-π, π) is

a. 0 b. 2π/3

c. 4π/3 d. 4π **Ans : a**

26. The function $f(x)=e^x$ is

a. Even b. Odd

c. Either even or odd d. Neither even nor odd **Ans : d**

27. The function $(1+x)^2$ is

a. Even b. Odd

c. Either even or odd d. None of these **Ans : d**

28. If the Fourier series of f(x) contain only sine terms then f(x) is

a. even function b. odd function

c. Either even or odd d. Neither even nor odd **Ans : b**

29. If the Fourier series of f(x) contain only cosine terms then f(x) is

a. even function b. odd function

c. Either even or odd d. Neither even nor odd **Ans : a**

30. The value of the constant a_0 in the Fourier expansion of $f(x) = x - x^2$, in (-π, π) is

a. $\pi - \pi^3$ b. $\pi^2/2 - \pi^4/4$

c. $\pi/2 - \pi^4/4$ d. 0 **Ans : d**

31. The value of the Fourier series of $f(x) = \sqrt{1-\cos x}$ in (0, 2π) at x = 0 is

a. 2π b. 0

c. $\sqrt{2}$ d. 2 **Ans : b**

32. The value of the constant a_0 in the Fourier expansion of $f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$

a. π b. 2π

c. 3π d. 4π **Ans : a**

33. The value of the b_n in the Fourier expansion of $f(x) = |x|$, in (-π, π) is

a. π b. 2π

c. 3π d. 0 **Ans : d**

34. The Fourier series of $f(x) = x \sin x$, in (-π, π) contain

a. only sine terms b. only cosine terms

c. both cosine and sine terms d. none **Ans : b**

35. The Fourier series of $f(x) = |x|$, in (-π, π) contain

a. only sine terms b. only cosine terms

c. both cosine and sine terms d. none **Ans : b**

36. To which value, the Fourier series of $f(x) = \begin{cases} l - x, & 0 < x < l \\ 0, & l < x < 2l \end{cases}$ converges at x = 0

a. l b. $l/3$

c. $l/2$ d. $2l$ **Ans : a**

37. The value of a_0 in the Fourier series of $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ is

a. π b. 2π

c. 3π d. $\pi/2$ **Ans : d**

38. The value of the constant a_0 in the Fourier expansion of $f(x) = (x-1)^2,\ 0 < x < 1$ is

a. $2/3$ b. $1/3$

c. $4/3$ d. $8/3$ **Ans : a**

39. The value of the constant a_n in the Fourier expansion of $f(x) = e^x,\ 0 < x < \pi$ is

a. 2π b. $2/\pi$

c. $1/\pi$ d. 0 **Ans : d**

40. The value of the constant a_n in the Fourier expansion of $f(x) = \cos x,\ 0 < x < \pi$ is

a. 2π b. $2/\pi$

c. $1/\pi$ d. 0 **Ans : d**

41. The value of the b_n in the Fourier expansion of unity in $(0, \pi)$ for even values of "n" is

a. 2π b. $2/\pi$

c. $1/\pi$ d. 0 **Ans : d**

42. If x_0 is a continuous point, then the Fourier series of f(x) converges to

a. $f(x)$ b. $f(x_0)$

c. $f(0)$ d. 0 **Ans : b**

43. The Fourier series of f(x) in a < x < b converges to --------------- at x = 0

a. $f(a)$ b. $f(b)$

c. $\dfrac{f(a)+f(b)}{2}$ d. 0 **Ans : c**

44. The value of the b_n in the Fourier expansion of $f(x) = x\sin x,\ -\pi < x < \pi$ is

a. 2π b. $\pi/3$

c. $1/\pi$ d. 0 **Ans : d**

45. The value of the a_n in the Fourier expansion of $f(x)$ *in* $-3 < x < 3$ is

a. $\dfrac{2}{3}\int_0^3 f(x)\cos\dfrac{n\pi x}{3}\,dx$ b. $\dfrac{1}{3}\int_0^3 f(x)\cos nx\,dx$

c. $\dfrac{2}{3}\int_{-3}^3 f(x)\cos\dfrac{n\pi x}{3}\,dx$ d. $\dfrac{1}{3}\int_{-3}^3 f(x)\cos nx\,dx$ **Ans : d**

46. The period of f(x) to expanded as a Fourier series in $-\pi < x < \pi$

a. 0 b. π

c. 2π d. 3π **Ans : c**

PART A
TWO MARKS

1. Explain periodic function with two examples.

Solution: A function f(x) is said to have a period T if for all x, $f(x+T) = f(x)$,

Where T is a positive constant. The least value of $T > 0$ *is called the period of* $f(x)$.

For examples, $f(x) = \text{Sin}x$

$f(x+2\pi) = \text{Sin}(x+2\pi) = \text{Sin}x$

Here, $f(x) = f(x+2\pi)$

2. State Dirichlet's condition for a given function to expend in Fourier series.

Solution: Any function f(x) can be developed as a Fourier series, provided

i) f(x) is periodic, single valued & finite.
ii) f(x) has a finite number of discontinuities in any one period
iii) f(x) has a finite number of maxima and minima

3. State general Fourier series.

solution: The Fourier series of f(x) in $c \le x \le c + 2l$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

Where a_0, an &b_n are called Fourier coefficients(or) Euler constants

4. Find the coefficient of b_n of $\cos 5x$ in the Fourier cosine series of the function

$f(x) = \sin 5x$ in the in the interval $(0, \pi)$.

Solution: The Fourier Cosine series is

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} \cos 5x \cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}[\cos(5+n)x + \cos(5-n)x]d$$

$$= \frac{2}{\pi}\left[\frac{\sin(5+n)x}{5+n} + \frac{\sin(5-n)x}{5-n}\right]_0^{\pi} = 0; \text{Therefore}, b_n = 0$$

5. Find the constant a_0 of the Fourier series for the function of $f(x) = x$ in $0 \le x \le 2\pi$

Solution:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx = \frac{1}{\pi}\int_0^{2\pi} xdx = \frac{1}{\pi}\left(\frac{x^2}{2}\right)_0^{2\pi} = 2\pi$$

6. Obtain the first term of the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^{\pi} = \frac{2}{3}\pi^2$$

7. If $f(x) = x^3, -\pi < x < \pi$. *Find the constant term of its Fourier series.*

Solution:

$f(x) = x^3$, is an odd function. Therefore, the fourier constants $a_0 = 0$

8. Find the constant term in the Fourier series expansion of $f(x) = x$ in $(-\pi, \pi)$

Solution:

$a_0 = 0$ since $f(x)$ is an odd function $(-\pi, \pi)$

9. Find the sum of the Fourier series of $f(x) = x + x^2$ in $-\pi < x < \pi$ at $x = \pi$.

Solution:

Given $f(x) = x + x^2$

The sum of Fourier series is equal to the arithmetic mean of the value of $f(x)$ at $x = \pi$ and $x = -\pi$.

Sum of Fourier series $= \dfrac{f(\pi) + f(-\pi)}{2} = \dfrac{\pi + \pi^2 - \pi + \pi^2}{2} = \pi^2$

10. What is the constant term a_0 and the coefficient of cosnx, a_n in the Fourier series of $f(x) = x - x^3$ in $(-\pi, \pi)$.

Solution:

$$f(x) = x - x^3 \Rightarrow f(-x) = -x + x^3$$

$$= -(x - x^3) = -f(x)$$

Therefore, f(x) is an odd function of x in (-π, π). Therefore, the Fourier series of f(x) Contains sine terms only. Therefore, $a_0 = 0$ and $a_n = 0$

11. Find b_n in the expansion of x^2 as a Fourier series in $(-\pi, \pi)$.

Solution:

$f(x) = x^2 \Rightarrow f(-x) = x^2 = f(x)$

Therefore, $f(x)$ is an even function of x in $(-\pi, \pi)$. The coefficient b_n of Sinnx in the

Fourier expansion is zero. Therefore, $b_n = 0$

12. Find a_n in expanding e^{-x} as Fourier series in $(-\pi, \pi)$.

Solution:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x} \cos \quad nx\, dx = \frac{1}{\pi}\left[\frac{e^{-x}}{1+n^2}(-\text{Cos } nx + n \text{ Sin } nx)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-\pi}(-1)^n + (-1)^n e^{\pi}\right]$$

$$a_n = \frac{(-1)^n (e^{\pi} - e^{-\pi})}{\pi(1+n^2)} = \frac{2(-1)^n \text{Sinh } \pi}{\pi(1+n^2)}$$

13. Fine the Fourier constant b_n for x Sin x in $(-\pi, \pi)$.

Solution:

Let $f(x) = x \operatorname{Sin} x$, Therfore, $f(x) = (-x)\operatorname{Sin}(-x) = x \operatorname{Sin} x = f(x)$

Therefore, $f(x)$ is even function of x in $(-\pi, \pi)$.

Therefore, $b_n = 0$

14. If $f(x) = |x|$ is expanded as a Fourier series in $(-\pi, \pi)$, find the value of a_n?

Solution:

$f(x) = |x|$ is an odd function in $(-\pi, \pi)$.

Therefore, the value of the Fourier coefficient $a_n = 0$.

15. Suppose the function x cos x has the series expansion $\sum_{n=1}^{\infty} b_n$ Sin x in $(-\pi, \pi)$, find the value of b_1.

Solution: $b_1 = \frac{2}{\pi}\int_0^{\pi} x\operatorname{Cos}x\operatorname{Sin}x\,dx = \frac{1}{\pi}\int_0^{\pi} x\operatorname{Sin}2x\,dx = \frac{1}{\pi}\left[x\left(\frac{-\cos 2x}{2}\right) + \left(\frac{\operatorname{Sin}2x}{4}\right)\right]_0^{\pi}$

$$= \frac{1}{\pi}\left(\frac{-\pi}{2}\right) = \frac{-1}{2}$$

16. Find the value of a_n in the Fourier expansion of f(x)= x^2 in $(0, 2\pi)$

Solution:

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \operatorname{Cos}nx\,dx = \frac{1}{\pi}\left[x^2\left(\frac{\operatorname{Sin}nx}{n}\right) - (2x)\left(\frac{-\operatorname{Cos}nx}{n^2}\right) + 2\left(\frac{\operatorname{Sin}nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left(\frac{4\pi}{n^2}\right) = \frac{4}{n^2}$$

17. Does f(x) = tan x possess a Fourier expansion in (0, π).

Solution:

f(x) =tan x has an infinite discontinuity at $x = \frac{\pi}{2}$

Since, the Dirichlet`s conditions on continuity is not satisfied, the function f(x)=tan x has no Fourier expansion.

19. When an even function f(x) is expanded in a Fourier series in the interval from $-\pi$ to π. Show that b_n=0

Solution:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \text{Sinnx}\ dx$$

Since f(x) is even and Sin nx is odd then product f(x) Sin nx is an odd function.

By Property of definite integral b_n =0.

20. If f(x) is an odd function defined in (-1,1) What are the values of a_0 and a_n

Solution:

$$a_0 = 0 \text{ and } a_n = 0 \text{ since } f(x) \text{is an odd function.}$$

FOURIER TRANSFORM

PART A(TWO MARKS)

1. **Find the Fourier Transform of** $f(x) = \begin{cases} 1 & \text{in} \quad |x| < a \\ 0 & \text{in} \quad |x| > a \end{cases}$

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{isx}dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} f(x)e^{isx}dx + \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} f(x)e^{isx}dx + \frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} f(x)e^{isx}dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} f(x)e^{isx}dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} 1.e^{isx}dx = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{isx}}{is}\right]_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{is}\left(e^{isa} - e^{-isa}\right)$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{is} 2i \sin sa = \sqrt{\frac{2}{\pi}} \frac{\sin sa}{s}$$

2.Find the Fourier sine transform of e^{-x}

The Fourier sine transform of f(x) is given by

$$F_S[f(x)] = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} f(x) \sin sx dx$$

$$\text{Here} \quad e^{-x} = e^{-|x|} \quad \text{for} \quad x > 0$$

$$\therefore F_S\left[e^{-x}\right] = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} e^{-x} \sin sx dx$$

$$= \sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2 + 1}\right)$$

3. Find the Fourier sine transform of e^{ax} .

$$F_S\left[e^{ax}\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx$$

$$=\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-ax}\sin sx\,dx$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+a^2}\right)$$

4. Find the Fourier cosine transform of e^{-x}

$$F_C\left[e^{-x}\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx$$

$$=\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-x}\cos sx\,dx$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{s^2+1}\right) \qquad \left[\because \int_0^\infty e^{-ax}\cos bx\,dx=\frac{a}{a^2+b^2}\right]$$

5. State the convolution theorem for Fourier transforms

If F(s) and G(s) are the Fourier transform of f(x) and g(x) respectively then the fourier transform of the convolution of f(x) and g(x) is the product of their Fourier Transform.

$$\text{ie}\quad F\left[(f*g)(x)\right]=F(s).G(s)$$

$$\text{ie}\quad \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(f*g)(x)e^{isx}dx=\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{isx}dx\right\}.\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}g(x)e^{isx}dx\right\}$$

6. Write the Fourier Transform pair.

If f(x) is a given function, then $F\left[f(x)\right]$ **and** $F^{-1}\left[F(f(x))\right]$ **are called Fourier transform pair, where**

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-isx} dx$$

$$F^{-1}[F(f(x))] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(f(x)) e^{isx} ds$$

7. Find the Fourier sine transform of $\frac{1}{x}$

$$F_S[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{1}{x} \sin sx dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{\sin sx}{x} dx$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{\pi}{2} = \sqrt{\frac{\pi}{2}} \qquad \left[\because \int_0^{\infty} \frac{\sin ax}{x} dx = \frac{\pi}{2}, a > 0 \right]$$

8.Write down the Fourier cosine transform pair of formulae.

Fourier cosine transform of f(x) is

$$F_C[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx dx$$

Inverse Fourier cosine transform is

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx ds$$

9. Write down the Fourier sine transform pair.

Fourier sine transform of f(x) is

$$F_S[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\sin sx\,dx$$

Inverse Fourier cosine transform is

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_S[f(x)]\sin sx\,ds$$

10. Define self reciprocal function and give example.

If the transform of f(x) is equal to f(s),then the function f(x) is called self-reciprocal.Example

$f(x) = e^{-\frac{x^2}{2}}$ **is self reciprocal under Fourier cosine transform.**

11.Give a function which is self reciprocal with respect to the Fourier sine transform.

$f(x) = e^{-\frac{x^2}{2}}$ **is self reciprocal under Fourier sine transform.**

12. State Parseval's identity on complex Fourier Transforms.

$$\int_0^\infty |f(x)|^2 dx = \int_0^\infty |F(s)|^2 \; ds$$

PART C
MODEL QUESTIONS

1. Find the Fourier series for $f(x) = |\cos x|$ in the interval $(-\pi, \pi)$.

2. Find the Fourier series for $f(x) = \begin{cases} l - x & in \ \ 0 \le x \le l \\ 0 & in \ \ l \le x \le 2l \end{cases}$. Hence deduce the sum to infinity of the series $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$.

3. Obtain the Fourier series for $f(x)$ of period 2l and defined as follows

$$f(x) = \begin{cases} L + x & \text{in } (-L, 0) \\ L - x & \text{in } (0, L) \end{cases} \quad \text{Hence deduce that } \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$

4. Show that for 0 < x < l, $x = \frac{l}{2} - \frac{4l}{\pi^2}\left(\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \cdots\right)$. Deduce that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \frac{\pi^4}{96}$.

5. Find the Fourier series for the function $f(x) = \begin{cases} x & \text{in } \ 0 < x < 1 \\ 1-x & \text{in } \ 1 < x < 2 \end{cases}$

6. Find the Fourier series of $f(x) = x^2$ in $-\pi < x < \pi$ Hence find ,

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$

7. Find the Fourier series for the function

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & \text{in } \ -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & \text{in } \ 0 < x < \pi \end{cases}$$

8. Find the Fourier series for the function f(x) =x sin x , $0 < x < 2\pi$ and hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} \dots = \frac{\pi-2}{4}$

9. Find the Fourier series for the function $f(x) = x(\pi^2 - x^2)$ in $(-\pi, \pi)$.

10. Find the Fourier series expansion of period 2L for the function

$f(x) = (L - x)^2$ in the range (0, 2L). Deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

11. Find the Fourier series of f(x) = $\begin{cases} -K & \text{in } (-\pi, 0) \\ K & \text{in } (0, \pi) \end{cases}$

12. Obtain Fourier series for f(x) of period 2L and defined as follows:

f(x) = $\begin{cases} L - x & \text{in } (0, L) \\ 0 & \text{in } (L, 2L) \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

13. Determine the Fourier series expansion of f(x)= $\left(\frac{\pi - x}{2}\right)^2$ in $0 < x < 2\pi$ with period 2π.

14.Find the Fourier Transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence find $\int_0^{\infty} \frac{\sin x}{x} dx$

15. Find the Fourier Transform of $e^{\frac{-x^2}{2}}$

16. Verify the convolution theorem under Fourier Transform for $f(x) = g(x) = e^{-x^2}$

17. Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$ using Parseval's identity.

18. Find the Fourier Transform of $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence find $\int_0^{\infty} \frac{\sin^4 t}{t^4} dt$

19. Find the Fourier Transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ and hence deduce that

$$\int_0^\infty \frac{\sin t - t\cos t}{t^3} dt = \frac{\pi}{4}$$

20. Find the Fourier cosine and sine transforms of $f(x) = e^{-ax}, a > 0$ and hence deduce the inverse formula.

21. Find the Fourier cosine transform of $e^{-a^2x^2}$,a>0. Hence show that the function $e^{\frac{-x^2}{2}}$

22. Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$

23. Derive the Parseval's identity for Fourier Transforms.

24. State and prove convolution theorem on Fourier Transform.

25. Find the Fourier sine and cosine transform of x^{n-1} and hence prove $\frac{1}{\sqrt{x}}$ is self reciprocal under Fourier sine and cosine transforms.