

UNIT-III
PARTIAL DIFFERENTIAL EQUATIONS

PART-A

(TWO MARKS)

1. Find the solution of $px^2 + qy^2 = z^2$

Ans: The S.E is $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$

Taking I two members ,we get

$$\frac{dx}{x^2} = \frac{dy}{y^2}$$

Integrating we get

$$\frac{-1}{x} = \frac{-1}{y} + c_1$$

$$u\left(\frac{1}{y} - \frac{1}{x}\right) = c_1$$

Taking last two members ,we get

$$\frac{dy}{y^2} = \frac{dz}{z^2}$$

Integrating we get

$$\frac{-1}{y} = \frac{-1}{z} + c_2$$

$$v\left(\frac{1}{z} - \frac{1}{y}\right) = c_2$$

The complete solution is

$$\phi\left(\frac{1}{y} - \frac{1}{x}, \frac{1}{z} - \frac{1}{y}\right) = 0$$

2. Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$

Ans: The complete solution is $z = ax + by + a^2 - b^2$ (1)

$$\text{Now } \left. \begin{aligned} \frac{\partial z}{\partial a} = x + 2a = 0 \Rightarrow a = \frac{-x}{2} \\ \frac{\partial z}{\partial b} = y - 2b = 0 \Rightarrow b = \frac{y}{2} \end{aligned} \right\} \dots\dots\dots(2)$$

23MAT103-DIFFERENTIAL EQUATIONS AND TRANSFORMS

Sub (2) in (1), we get

$$\begin{aligned}z &= \frac{-x^2}{2} + \frac{y^2}{2} + \frac{x^2}{4} - \frac{y^2}{4} \\ &= \frac{-x^2}{4} + \frac{y^2}{4}\end{aligned}$$

$$y^2 - x^2 = 4z \text{ which is S.I}$$

3. Write the complete solution of $p+q=x+y$

Ans: Let $p+q=x+y=k$
 $p-x=k$, $y-q=k$
 $p=k+x$ $q=y-k$
 $z = \int p dx + \int q dy$
 $z = \int (k+x) dx + \int (y-k) dy$
 $= \frac{(k+x)^2}{2} + \frac{(y-k)^2}{2} + c$

4. Find the partial integral of $(D^2 - 2DD' + D'^2)z = e^{x-y}$

Ans: P.I = $\frac{1}{D^2 - 2DD' + D'^2} e^{x-y}$
 $= \frac{1}{4} e^{x-y}$

5. Find the complete integral of $q=2px$

Ans: $q=2px$
 $xp = \frac{q}{2} = k$
 $p = \frac{k}{x}$, $q = 2k$
 $z = \int p dx + \int q dy$
 $z = k \log x + 2ky + c$

6. Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$

Ans: A.E is $(m^2 - 3m + 2) = 0$
 $(m-1)(m-1)(m+2) = 0$
 $m = 1, 1, -2$
 $z = f_1(y+x) + xf_2(y+x) + f_3(y-2x)$

7. Find the complete solution of the PDE $\sqrt{p} + \sqrt{q} = 1$

Ans: Given $\sqrt{p} + \sqrt{q} = 1$ (1)
 Let $z = ax+by+c$ (2) be the solution of (1)

$$\left. \begin{aligned} \frac{\partial z}{\partial x} = p = a \\ \frac{\partial z}{\partial y} = q = b \end{aligned} \right\} \text{.....(3)}$$
 Sub (3) in (1)
 $\sqrt{a} + \sqrt{b} = 1$ (4)
 $z = ax+by+c$ is a solution of (1)
 From (4) $\sqrt{a} = 1 - \sqrt{b}$ (5)
 Sub (5) in (2)
 $z = (1 - \sqrt{b})^2 x + by + c$ which is complete integral

8. Find the Complete integral of the partial differential equation $z = px + qy + p^2 + q^2$

Ans: Sub $p = a$ and $q = b$
 $z = ax + by + a^2 + b^2$

9.Solve $(D^2 - 3DD' + 2D'^2)z = 0$

Ans: A.E is $m^2 - 3m + 2 = 0$
 $m=1,2$
 $z = f_1(y + x) + f_2(y + 2x)$

9. Solve $(D^3 - 2D^2D')z = 0$

Ans: A.E is $m^3 - 2m^2 = 0$
 $m=0,0,2$
 $z = f_1(y) + xf_2(y) + f_3(y + 2x)$

10.Find the complete solution of $pq=xy$

Ans: $pq=xy$
 $\frac{p}{x} = \frac{y}{q}$

$$\begin{aligned} \text{Let } \frac{p}{x} &= \frac{y}{q} = k \\ p &= kx, q = \frac{y}{k} \\ z &= \int p dx + \int q dy \\ &= \int kx dx + \int \frac{y}{k} dy \\ &= k \frac{x^2}{2} + \frac{1}{k} \frac{y^2}{2} + c \end{aligned}$$

11.Solve $\frac{\partial^2 z}{\partial x^2} - \frac{4\partial^2 z}{\partial x \partial y} + \frac{4\partial^2 z}{\partial y^2} = 0$

Ans: A.E is $m^2 - 4m + 4 = 0$
 $(m - 2)^2 = 0$
 $m = 2, 2$
 $z = f_1(y + 2x) + 2f_2(y + 2x)$

12.Write Particular integral of $\frac{\partial^2 z}{\partial x^2} - \frac{5\partial^2 z}{\partial x \partial y} + \frac{6\partial^2 z}{\partial y^2} = e^{x+y}$

Ans: P.I = $\frac{1}{D^2 - 5DD' + 6D'^2} e^{x+y}$ Sub $D = 1$ and $D' = 1$
 $= \frac{1}{1 - 5 + 6} e^{x+y}$
 $= \frac{1}{2} e^{x+y}$

13.Write Particular integral of $(D^2 + 3DD' + 2D'^2)z = \sin(x + 5y)$

Ans: P.I = $\frac{1}{D^2 + 3DD' + 2D'^2} \sin(x + 5y)$ Sub $D^2 = -1, DD' = -5$ and $D'^2 = -25$
 $= \frac{1}{-1 - 15 - 50} \sin(x + 5y)$
 $= \frac{1}{-66} \sin(x + 5y)$

14.Find the singular Integral of $z = px + qy + p^2$

Ans: Complete solution is $z = ax + by + a^2$ (1)
 $\frac{\partial z}{\partial a} = x + 2a = 0$ (2)
 $\frac{\partial z}{\partial b} = y = 0$ (3)
 From (2) we get

$$x = -2a \text{ (or) } a = -x/2 \dots\dots (4)$$

Sub (3) and (4) in (1), we get

$$z = \frac{-x^2}{2} + \frac{x^2}{4} = \frac{-x^2}{4}$$

$$4z = -x^2$$

15. Write Particular integral of $(D^2 - DD')z = \sin(x + y)$

Ans:

$$\begin{aligned} \text{P.I} &= \frac{1}{D^2 - DD'} \sin(x + y) \text{ Sub } D^2 = -1, DD' = -1 \\ &= \frac{1}{-1+1} \sin(x + y) \\ &= \frac{x}{2D} \sin(x + y) \\ &= \frac{-x}{2} \cos(x + y) \end{aligned}$$

24 Write Particular integral of $(D^2 + 2DD' + D'^2)z = e^{(x-y)}$

Ans:

$$\begin{aligned} \text{P.I} &= \frac{1}{D^2 + 2DD' + D'^2} e^{(x-y)} \text{ Sub } D = 1, D' = -1 \\ &= \frac{1}{1 - 2 + 1} e^{(x-y)} \\ &= \frac{x}{2D + 2D'} e^{(x-y)} \\ &= \frac{x^2}{2} e^{(x-y)} \end{aligned}$$

25. Find the complete solution of $p-q=0$

Ans: $p-q=0$
 This is of the type $F(p,q)=0 \dots\dots (1)$
 Let $z=ax+by+c \dots\dots (2)$ be the solution of PDE
 From (2) we get $p=a, q=b \dots\dots (3)$
 Sub (3) in (1), we get
 $a-b=0 \Rightarrow a=b \dots\dots\dots (4)$
 Sub (4) in (2), we get $z = a(x+y)+c$

UNIT-III
PARTIAL DIFFERENTIAL EQUATIONS
PART-C

1.	Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$	
2.	Solve $(D^2 + DD' - 6D'^2)z = y \cos x$	
3.	Solve $z = px + qy + \sqrt{p^2 + q^2 + 1}$	
4.	Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(2x + y)$	
5.	Find the singular integral of $z = px + qy + p^2 + pq + q^2$	
6.	Solve the PDE $(x - 2z)p + (2z - y)q = y - x$	
7.	Solve $(D^2 + 3DD' - 4D'^2)z = \cos(2x + y) + xy$	
8.	Solve $(D^2 - DD' + 2D)z = e^{2x+y} + 4$	
9.	Solve $(mz - ny)p + (nx - lz)q = ly - mx$	
10.	Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$	
11.	Solve $p(1+q)=qz$	
12.	Solve $(D^2 - DD' - 30D'^2)z = xy + e^{2x+y}$	
13.	Solve $(3z - 4y)p + (4x - 2z)q = 2y - 3x$	
14.	Solve $(D^2 + 4DD' - 5D'^2)z = e^{2x-y} + \sin(x - 2y)$	
15.	Solve $z = 1 + p^2 + q^2$	
16.	Solve $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+y}$	
17.	Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$	
18.	Solve $(D^3 + D^2D' - 4DD'^2 - 4D'^3)z = \cos(2x + y)$	
19.	Solve $(2D^2 - DD' - D'^2 + 6D + 3D')z = xe^y$	
20.	Solve $(x - y)p + (y - x - z)q = z$	
21.	Solve $(1 + y)p + (1 + x)q = z$	

23MAT103-DIFFERENTIAL EQUATIONS AND TRANSFORMS

22.	Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$	
23.	Solve $(D^2 + 4DD' - 5D'^2)z = x + y^2 + \pi$	
24.	Solve $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$	
25.	Solve $(D^3 - 4D^2D' + 4DD'^2)z = 6\sin(3x + 6y)$	
26.	Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = \cos 2x \cos y$	