Large Samples

Test of Significance of Large Samples:

Sample Size $n > 30$

Test of Significance for single Mean

Null Hypothesis: $H_0 : \mu = \mu_0$

Alternative Hypothesis: $H_1 : \mu \neq \mu_0$

(or) : $\mu > \mu_0$

(or) : $\mu < \mu_0$

Test Statistic: $z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$ ($\underline{SD}$ σ is known)

(or) $z = \dfrac{\bar{x} - \mu}{(s/\sqrt{n})}$ (S.D $= \sigma$ is not known)

Problem:

A sample of 900 members has a mean of 3.4 cms and S.D. 2.61 cms. Is the sample from a large population of mean 3.25 cm and S.D. 2.61 cms. If the populations is normal and its mean is unknown find the 95% fiducial limits of true mean.

Solution:

Given: $n = 900$ members | $\bar{x} = 3.4$

$\sigma = 2.61$ cms | $\mu = 3.25$

$n > 30$ large sample.

S.D $= s = 2.61$ cms

Null Hypothesis: $H_0 : \mu = 3.25$.

Sample is drawn from population with $\mu = 3.25$

Alternative Hypothesis $H_1 : \mu \neq 3.25$ (Two-tailed test)

The test statistic is

$$z_{cal} = \frac{\bar{x} - \mu}{\left(s/\sqrt{n}\right)} = \frac{3.4 - 3.25}{\left(\frac{2.61}{\sqrt{900}}\right)} = 1.724$$

$\therefore z_{cal} = 1.724$

<u>Level of Significance</u>: $\alpha = 5\% = 0.05$

Critical Value: $z_{tab} = 1.96$ (5% LoS)

Conclusion: $|z| = 1.724 < 1.96 = z_{tab}$

$$z_{cal} < z_{tab}$$

$\therefore$ we accept the null hypothesis H_0.

$\therefore$ The sample is taken from population whose mean is 3.25 cm.

Confidence limits:

$$\left.\begin{array}{ll} 95\% \text{ fiducial limits} & \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} \\ 99\% \text{ fiducial limits} & \bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}} \end{array}\right\} \text{ — } (*)$$

95% fiducial limits $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

$$= 3.4 \pm 1.96 \left(\frac{2.61}{\sqrt{900}}\right)$$

$$= 3.4 \pm 0.1705$$

$$= 3.57 \text{ and } 3.2295$$

Problem-2:

A random sample of 50 observations from the normal population gave an arithmetic mean of 32 units with a standard deviation of 2 units. Test whether the population mean is 30 at 1% level.

Solution:

$n = 50$ | $\mu = 30$

$\bar{x} = 32$ | $L.o.S = 1\%$

$S.D = 2 = \sigma = s$

Null Hypothesis: $H_0 : \mu = 30$

The population mean is 30 only.

Alternative Hypothesis:

$H_1 : \mu \neq 30$ (Two-tailed)

Test Statistic:

$$z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})} = \frac{32 - 30}{(2/\sqrt{50})}$$

$$= \frac{2}{2/\sqrt{50}} = 7.07$$

$$z_{cal} = 7.07$$

$LoS = 1\%$ $\alpha = 0.01$

critical value $z_{tab} = 2.58$ (1% LoS)

Here $z_{cal} > z_{tab}$.

$\therefore$ We reject the Null Hypothesis.

$\therefore$ the population mean is not 30 only.

3) The mean height of college students in a city are normally distributed with S.D 6 cms. A sample of 100 students has mean height 158 cms. Test the hypothesis that the mean height of college students in the city is 160 cms. Also obtain 99% confidence limits for the true mean.

Solution:

$n = 100$ (large sample)

$\sigma = 6$ cms

$\bar{x} = 158$

$\mu = 160$

Null Hypothesis: $H_0 : \mu = 160$

Mean height of college students is 160 cms.

Alternative Hypothesis: $H_1 : \mu \neq 160$ (two-tailed test)

Level of Significance: $\alpha = 1\% = 0.01$

Test statistic: $z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})} = \dfrac{158 - 160}{(6/\sqrt{100})}$ $z_{cal} = -3.3$

$$|z|_{cal} = 3.33$$

Critical Value:

$z_{0.01} = 2.58$ $\quad z_{tab} = 2.58$ (1% LOS)

$$z_{cal} > z_{tab}$$

∴ We reject the null Hypothesis

∴ The mean height of college students is not 160 cms.