Test of Significance for Single Proportion:

Test Statistic: $$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

p - sample proportion

P - population proportion

n - sample size.

$Q = 1 - P$.

Problem:

A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% LoS.

Solution:

Given: $n = 200$

No. of pieces good $= 200 - 18$

$= 182$.

p - sample proportion $= \frac{182}{200} = 0.91$

P - population proportion $= \frac{95}{100} = 0.95$.

$Q = 1 - P = 1 - 0.95 = 0.05$.

Null Hypothesis $H_0 : P = 95\%$.

No. of pieces good is 95%.

Alternative Hypothesis $H_1 : P < 0.95$ (left tail

Test statistic

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.91 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}}$$

$$= \frac{-0.04}{0.0154} = -2.59$$

$|Z_{cal}| = 2.59$

LoS 5%, left tailed, $Z_{tab} = 1.645$

$\therefore Z_{cal} > Z_{tab}$.

$\therefore$ We reject H_0, null hypothesis

$\therefore$ The manufacturer does not produce good products

i.e., his claim is rejected.

2) In a sample of 1000 people in Karnataka 540 rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% LoS?

Solution:

Given: $n = 1000$

p - sample proportion $= \frac{540}{1000}$

$p = 0.54$.

P - population proportion

$= \frac{1}{2} = 0.5$

$Q = 1 - 0.5 = 0.5$

Null Hypothesis: $H_0: P = 50\%$

Both rice and wheat eaters are equally popular

Alternative Hypothesis: $H_1: P \neq 50\%$. (Two-tailed)

Test statistic.

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5}{1000}}}$$

$$Z_{cal} = 2.532$$

LoS, 1%, two-tailed, $Z_{tab} = 2.58$.

$Z_{cal} < Z_{tab}$

∴ We accept H_0, i.e., both rice and wheat eaters are equally popular.

8) In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

Solution:

Given: $n = 600$

No. of smokers = 325

p - sample proportion $= \frac{325}{600} = 0.5417$

P - population proportion $= \frac{1}{2} = 0.5$

$Q = 1 - P = 1 - 0.5 = 0.5$

Null Hypothesis: $H_0 : P = 0.5$

No. of smokers and non-smokers are equal in the city.

Alternative Hypothesis: $H_1 : P > 0.5$ (Right tailed)

Test statistic

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.5417 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}}$$

$$Z_{cal} = 2.04$$

Los 5%, Right tailed, $Z_{tab} = 1.645$

$Z_{cal} > Z_{tab}$.

∴ we reject H_0, Null hypothesis.

∴ Smokers are higher number than non-smokers in the city.