Student t-Test for Difference of Means

* Null Hypothesis $H_0 : \mu_1 = \mu_2$

* Alternative Hypothesis $H_1 : \mu_1 \neq \mu_2$

(or) $\mu_1 > \mu_2$.

(or) $\mu_1 < \mu_2$

* Test statistic : $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

where $s^2 = \dfrac{\Sigma (x_1 - \bar{x}_1)^2 + \Sigma (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

(or) $s^2 = \dfrac{1}{n_1 + n_2 - 2}\left[n_1 s_1^2 + n_2 s_2^2\right]$

$s_1, s_2 \rightarrow$ sample standard deviation

* Degrees of freedom : $\gamma = n_1 + n_2 - 2$

1) The average number of articles produced by two machines per day are 200 and 250 with standard deviations 20 and 25 respectively on the basis of records of 25 days production. Can you regard both the machines equally efficient at 1% level of significance.

Solution:

Given:

$n_1 = 25$ $\quad n_2 = 25$

$\bar{x}_1 = 200$ $\quad \bar{x}_2 = 250$

$s_1 = 20$ $\quad s_2 = 25$

$$\therefore s^2 = \frac{1}{n_1+n_2-2}\left[n_1 s_1^2 + n_2 s_2^2\right]$$

$$= \frac{1}{25+25-2}\left[25(20)^2 + 25(25)^2\right]$$

$$= \frac{1}{48}\left[10000 + 15625\right] = 533.85$$

$$s^2 = 533.85$$

$$s = \sqrt{533.85}$$

$$\therefore \boxed{s = 23.10}$$

we know $n_1, n_2, \bar{x}_1, \bar{x}_2, s$

$\therefore$ we use student t-test.

*) Null Hypothesis: $H_0 : \mu_1 = \mu_2$.

Both machines are equally good.

*) Alternative Hypothesis: $H_1 : \mu_1 \neq \mu_2$.

*) Test statistic: $t_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{200-250}{23.10\sqrt{\frac{1}{25}+\frac{1}{25}}}$

$$= \frac{-50}{23.10(0.2828)} = \frac{-50}{6.53268}$$

$t_{cal} = -7.65$; $|t_{cal}| = 7.65$ //

*) degrees of freedom $n_1 + n_2 - 2$

$25 + 25 - 2$

$= 48$.

*) LOS = 1 %

*) t_{tab} at 1% with 48 dof

$t_{tab} = 2.58$.

(*) $t_{cal} > t_{tab}$.

$\therefore$ H_0 is rejected.

$\therefore$ both the machines are not equally efficient (good)

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2) Samples of two types of electric light bulbs were tested for length of life and following data were obtained.

	Type-I	Type-II
Sample No	$n_1 = 8$	$n_2 = 7$
Sample Means	$\bar{x}_1 = 1234$ hrs	$\bar{x}_2 = 1036$ hrs
Sample SD	$s_1 = 36$ hrs	$s_2 = 40$ hrs

~~Is the difference in the means $\bar{x}_1$ and $\bar{x}_2$ are given and also sample standard deviation s_1 and s_2 are given we use~~.

Is the difference in the means sufficient to warrant that type I is superior to type II regarding length of life.

Solution:

2 samples given $\therefore$ use student t-test difference of means.

*) Null Hypothesis $H_0 : \mu_1 = \mu_2$

The two types of bulbs are same.

*) Alternative Hypothesis $H_1 : \mu_1 > \mu_2$ (one-tailed)

*) Test statistic: $t_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Here,

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left[n_1 s_1^2 + n_2 s_2^2\right]$$

$$= \frac{1}{8 + 7 - 2}\left[8(36)^2 + 7(40)^2\right] = 1659.08$$

$$\therefore t_{cal} = \frac{1234 - 1036}{\sqrt{1659.08\left(\frac{1}{8} + \frac{1}{7}\right)}}$$

$$\boxed{t_{cal} = 9.39}$$

*) Degrees of freedom $\gamma = n_1 + n_2 - 2 = 8 + 7 - 2 = 13$.

LoS $= 5\%$

t_{tab} at 5% with 13 dof $= 1.77$

$$\boxed{t_{tab} = 1.77}$$

*) $t_{cal} > t_{tab}$.

$\therefore$ We reject null hypothesis H_0

*) $\therefore$ The two types I and II of electric bulbs are not Identical (same).

$\therefore$ type I is superior to type II.

3) Two horses A and B were tested according to the time (in seconds) to run a particular track with the following results.

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	27	-

Test whether you can discriminate b/w two horses. You can use the fact that 5% value of t for 11 degrees of freedom is 2.2.

Solution:

*) Two samples given $\therefore$ we use student t-test difference of means. Here $n_1 = 7$ | $n_2 = 6$

*) Null Hypothesis $H_0 : \mu_1 = \mu_2$

There is no significant difference b/w two horses.

*) Alternative Hypothesis: $H_1 : \mu_1 \neq \mu_2$

sample Means and SD's

x	$d_1 = x_i - \bar{x}$ $= x_i - 31.3$	d_1^2	y	$d_2 = y_i - \bar{y}$ $= y_i - 27.8$	d_2^2
28	-3.3	10.89	29	1.2	1.44
30	-1.3	1.69	30	2.2	4.84
32	0.7	0.49	30	2.2	4.84
33	1.7	2.89	24	-3.8	14.44
33	1.7	2.89	27	-0.8	0.64
29	-2.3	5.29	27	-0.8	0.64
34	2.7	7.29			
$\Sigma x = 219$		$\Sigma d_1^2 = 31.43$	$\Sigma y = 167$		$\Sigma d_2^2 = 26.84$

$\bar{x} = \frac{\Sigma x}{n_1} = \frac{219}{7}$ $\bar{y} = \frac{\Sigma y}{n_2} = \frac{167}{6}$

$\boxed{\bar{x} = 31.3}$ $\boxed{\bar{y} = 27.8}$

*) test statistic :

$$t_{cal} = \frac{\bar{x} - \bar{y}}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Here $s^2 = \frac{\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2}{n_1 + n_2 - 2}$

$$= \frac{31.43 + 26.84}{7 + 6 - 2} = \frac{58.27}{11} = 5.29$$

$\therefore s = \sqrt{5.29}$ $\therefore \boxed{s = 2.3}$

$$\Rightarrow t_{cal} = \frac{31.3 - 27.8}{2.3\sqrt{\frac{1}{7} + \frac{1}{6}}} = \frac{3.5}{2.3\sqrt{0.3074}}$$

$\boxed{t_{cal} = 2.73}$

*) Given: $t_{tab} = 2.2$ at 5% LoS & 11 dof.

$\therefore t_{cal} > t_{tab}$, we reject H_0 (Null Hyp)

Yes, $\therefore$ There is significant difference b/w two horses and they can be discriminated.

$$t_{cal} = \frac{\bar{x} - \bar{y}}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$