F-Test

- To test whethere if there is any significant difference b/w two estimates.
- If the two samples have come from the same population.

- Null Hypothesis : $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis — No

- Test statistic : $F = \frac{S_1^2}{S_2^2}$

where $S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1}$ n_1 - first sample size

$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1}$ n_2 - Second sample size.

and $S_1^2 > S_2^2$.

- Degrees of freedom :

$\gamma_1 = n_1 - 1$

$\gamma_2 = n_2 - 1$

(1) In one sample of 8 observations the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level.

Solution: Here its from same population. $\therefore$ we use F-Test.

Here $n_1 = 8$ | $n_2 = 10$.

$\Sigma(x-\bar{x})^2 = 84.4$ | $\Sigma(y-\bar{y})^2 = 102.6$

$$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} = \frac{84.4}{8-1} = \frac{84.4}{7}$$

$$S_1^2 = 12.057$$

$$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1} = \frac{102.6}{10-1} = \frac{102.6}{9}$$

$$S_2^2 = 11.4$$

Null Hypothesis:

$H_0 : S_1^2 = S_2^2$ There is no significant difference

Test Statistic: $F = \frac{S_1^2}{S_2^2}$

$$= \frac{12.057}{11.4}$$

$$= 1.057$$

$$\boxed{F_{cal} = 1.057}$$

5% LoS $\gamma_1 = n_1 - 1 = 7$ | $\gamma_2 = n_2 - 1 = 9$

F_{tab} at 5% LoS and (7,9) dof is 3.29 (i.e., $F_{tab} = 3.29$)

$\therefore F_{cal} < F_{tab}$, we accept the Null Hypothesis.

$\therefore$ we conclude that there is no significant difference b/w the two samples.

2) In comparing the variability of family income in two areas, a survey fielded the following data:

$n_1 = 100$ $\quad n_2 = 1100$

$s_1^2 = 25$ $\quad s_2^2 = 10$

where n_1 and n_2 are the sample sizes and s_1^2, s_2^2 are the sample variance of incomes for the two area respectively.

Assuming that the populations are normal test the hypothesis $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 > \sigma_2^2$ at $\alpha = 0.05$ level of significance.

<u>Solution</u>: Gn: $n_1 = 100$ $\quad n_2 = 1100$

$s_1^2 = 25$ $\quad s_2^2 = 10$.

since n_1, n_2, s_1^2 and s_2^2 are given, we use F-Test.

<u>Null Hypothesis</u>: ~~$H_0: \sigma_1^2 = \sigma_2^2$~~

$H_0: S_1^2 = S_2^2$.

There is no significant difference between the variances of two samples.

<u>Test statistic</u>:

$$F_{cal} = \frac{S_1^2}{S_2^2} = \frac{25}{10}$$

$$\therefore \boxed{F_{cal} = 2.5}$$

<u>Dof</u>:

$\gamma_1 = n_1 - 1 = 100 - 1 = 99$ | $\gamma_2 = n_2 - 1 = 110 - 1 = 109$

F_{tab} with $(99, 109)$ dof & 5% LoS is $F_{tab} = 1.35$

$F_{cal} > F_{tab}$ $\therefore$ we reject H_0

$\therefore$ There is significant difference.

3) Two random samples gave the following results.

Sample	Size	Sample Mean	Sum of squares of deviations from the mean.
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population.

Solution:

*) Two samples are given.

*) Sample Mean gn (use student t-test)

*) Sum of squares of deviations given (use F-test)

Given:

$n_1 = 10$ | $n_2 = 12$

$\bar{x}_1 = 15$ | $\bar{x}_2 = 14$

$\Sigma (x_1 - \bar{x}_1)^2 = 90$ | $(y_1 - \bar{y}_1)^2 = 108$

(*) F- Test [Equality of Variances].

Null Hypothesis: $H_0 : S_1^2 = S_2^2$.

The two samples are from the same normal population.

Test statistic: $F = \frac{S_1^2}{S_2^2}$

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{90}{10-1} = \frac{90}{9}$$

$$\boxed{S_1^2 = 10}$$

$$S_2^2 = \frac{\Sigma(y_1 - \bar{y}_1)^2}{n_2 - 1} = \frac{108}{12-1} = \frac{108}{11}$$

$$\boxed{S_2^2 = 9.82}$$

$$F_{cal} = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.018$$

$$\boxed{F_{cal} = 1.018}$$

F_{tab} at 5% los with (9, 11) dof

$\gamma_1 = n_1 - 1 = 9$ $\boxed{F_{tab} = 2.90}$

$\gamma_2 = n_2 - 1 = 11$

$\boxed{F_{cal} < F_{tab}}$ we accept the null Hypothesis.

i.e., the samples come from the same normal population.

(ii) Student t-test [equality of Means.

Null Hypothesis: $H_0 : \mu_1 = \mu_2$.

Alternative Hypothesis: $H_1 : \mu_1 \neq \mu_2$

Test statistic.

$$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \cancel{\frac{15 - 14}{3.15\sqrt{\frac{1}{10} + \frac{1}{12}}}}$$

$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(y_1 - \bar{y}_1)^2\right]$$

$$= \frac{1}{10 + 12 - 2}\left[90 + 108\right]$$

$$= 9.9$$

$$\boxed{S = 3.15}$$

$$\therefore t_{cal} = \frac{15-14}{3.15\sqrt{\frac{1}{10}+\frac{1}{12}}}$$

$$\boxed{t_{cal} = 0.74}$$

t_{tab} for 5% los with 20 dof is

$\nu = n_1 + n_2 - 2 = 10+12-2 = 20$

$$\boxed{t_{tab} = 2.086}$$

$\therefore t_{cal} < t_{tab}$

$\therefore$ We accept Null Hypothesis, i.e., the samples are from the same population.

Hence from (i) and (ii), the given samples are drawn from the same normal population.

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4) Two horses A and B were tested according to the time (in seconds) to run a particular track with the following results.

Horse A	28	30	32	33	33	29	34
Horse B.	29	30	30	24	27	29	–

Test whether the two horses have the same running capacity.

F-test.

Solution:

Given: $n_1 = 7$ | $n_2 = 6$

Null Hypothesis $H_0 : S_1^2 = S_2^2$

The two horses have the same running capacity.

Calculation of variances.

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
28	-3.28	10.75	29	0.8	0.64
30	-1.28	1.64	30	1.8	3.24
32	0.72	0.52	30	1.8	3.24
33	1.72	2.96	24	-4.2	17.64
33	1.72	2.96	27	-1.2	1.44
29	-2.28	5.20	29	0.8	0.64
34	2.72	7.40			
$\Sigma x = 219$		$\Sigma(x-\bar{x})^2 = 31.43$	$\Sigma y = 169$		$\Sigma(y-\bar{y})^2 = 26.84$

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{219}{7}$$

$$\boxed{\bar{x} = 31.28}$$

$$\Sigma(x-\bar{x})^2 = 31.43$$

$$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1}$$

$$= \frac{31.43}{7-1}$$

$$= \frac{31.43}{6}$$

$$= 5.238$$

$$\bar{y} = \frac{\Sigma y}{n_2} = \frac{169}{6}$$

$$\boxed{\bar{y} = 28.2}$$

$$\Sigma(y-\bar{y})^2 = 26.84$$

$$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1}$$

$$= \frac{26.84}{5}$$

$$= 5.368.$$

(*) Here $S_2^2 > S_1^2$

$\therefore$ Test statistic $F_{cal} = \frac{S_2^2}{S_1^2}$

$$= \frac{5.368}{5.238}$$

$\therefore$ $\boxed{F_{cal} = 1.02}$.

F_{tab} with 5% LoS (5, 6) degrees of freedom

$\nu_1 = n_1 - 1 = 7 - 1 = 6$

$\nu_2 = n_2 - 1 = 6 - 1 = 5$

$\boxed{F_{tab} = 4.39}$

$F_{cal} < F_{tab}$,

$\therefore$ We <u>accept</u> the H_0 (Null Hypothesis)

$\therefore$ The two horses have the same running capacity.