Chi-Square Test

(i) Goodness of fit.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

O - Observed frequency

E - Expected frequency

degrees of freedom $\gamma = n-1$.

Properties:

1) The mean of χ^2 is equal to the number of degrees of freedom.
2) The variance of χ^2 distribution is twice the degrees of freedom.
3) If χ^2 is a chi-square variate with γ degrees of freedom, then $\chi^2/2$ is a gamma variate with parameter $\gamma/2$
4) Standard χ^2 variate tends to standard normal variate as $n \to \infty$.

Applications:

1) To test if the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$.
2) To test the goodness of fit
3) To test the independance of attributes
4) To test the homogenity of independant estimates of the population variance.

Problem 1:

The following table gives the no. of aircraft accidents that occured during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Sol	Days	Mon	Tue	Wed	Thu	Fri	Sat
	No. of accidents	14	18	12	11	15	14

Solution:

Given: Total no. of accidents = 84

No. of days = 6.

Expected frequency $E = \frac{84}{6} = 14$.

Null Hypothesis: H_0: The accidents are uniformly distributed.

Alternative Hypothesis: H_1: The accidents are not uniformly distributed.

Level of Significance: LoS – 5%.

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
14	14	0	0
18	14	16	1.14
12	14	4	0.285
11	14	9	0.642
15	14	1	0.071
14	14	0	0

$$\leq \frac{(O-E)^2}{E} = 2.143$$

$$\chi^2_{cal} = \leq \frac{(O-E)^2}{E} = 2.143$$

Degrees of freedom = $d = n-1 = 6-1 = 5$

χ^2_{tab} at LoS – 5% and 5 d.f = 11.07

$\chi^2_{cal} < \chi^2_{tab}$.

$\therefore$ we accept the Null Hypothesis.

$\therefore$ the accidents are uniformly distributed over the week.

2) A die is thrown 264 times with the following results. Show that the die is biased

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	28	58	54	52

Solution:

Null Hypothesis H_0: The die is unbiased.

$$E = \frac{\Sigma E}{n} = \frac{264}{6} = 44.$$

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
40	44	16	0.3636
32	44	144	3.2727
28	44	256	5.8181
58	44	196	4.4545
54	44	100	2.2727
52	44	64	1.4545
$\Sigma O = 264$	$\Sigma E = 264$		$\Sigma \frac{(O-E)^2}{E} = 17.6362$

$$\chi^2 = \Sigma \frac{(O-E)^2}{E} = 17.6362.$$

Degrees of freedom $= \gamma = n - 1 = 5$.

χ^2_{tab} for 5 dof at 5% level $= 11.07$.

$\chi^2_{cal} > \chi^2_{tab}$. Hence the die is biased.

We reject null hypothesis