

chi-square Test for Independence of Attributes

Attributes \rightarrow quality or characteristics.

Gn:

A	a	b
B	c	d

Expected Frequencies:
 E_i 's

$(a+c)(a+b)/N$	$(b+d)(a+b)/N$	$a+b$
$(a+c)(c+d)/N$	$(b+d)(c+d)/N$	$c+d$
$a+c$	$b+d$	N

Degrees of freedom : $(r-1)(s-1)$

r - no. of rows

s - no. of columns.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Problem-1:

On the basis of information given below about the treatment of 200 patients suffering from a disease, state whether the new treatment is comparatively superior to the conventional treatment.

	Favourable	Non-favourable	Total
New	60	30	90
Conventional	40	70	110

Solution:

$$r = 2$$

$$s = 2$$

$$\text{Degrees of freedom} = (r-1)(s-1) = (2-1)(2-1) = 1 //$$

Expected frequencies: (E_i)

$\frac{(100)(90)}{200} = 45$	$\frac{(100)(90)}{200} = 45$	90
$\frac{(100)(110)}{200} = 55$	$\frac{(100)(110)}{200} = 55$	110
100	100	200

Null Hypothesis:

H_0 : No difference b/w the two treatments

Test Statistic:

$$\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$$

O	E	$(O-E)^2$	$\left(\frac{O-E}{E}\right)^2$
60	45	225	5
80	45	225	5
40	55	225	4.09
70	55	225	4.09
			18.18

$$\chi^2_{cal} = 18.18 //$$

χ^2_{tab} at 5% LOS 1 dof = 3.841

$\chi^2_{cal} > \chi^2_{tab}$, we reject the null hypothesis.

\therefore New and conventional treatments are not same.

2) The following table gives the classification of 100 workers according to gender and nature of work.

Test whether the nature of work is independent of the gender of the workers.

	Stable	Unstable	Total
Male	40	20	60
Female	10	30	40
	50	50	100

Solution:

$$r = 2 ; s = 2$$

$$\text{degrees of freedom} = (r-1)(s-1) = (2-1)(2-1) = 1$$

Expected Frequencies E_i 's.

$\frac{(50)(60)}{100} = 30$	$\frac{(50)(60)}{100} = 30$	60
$\frac{(50)(40)}{100} = 20$	$\frac{(50)(40)}{100} = 20$	40
50	50	100

Null Hypothesis

H_0 : Nature of work does not depend on gender of the workers.

Test Statistic:

$$\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$$

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
40	30	100	3.333
20	30	100	3.333
10	20	100	5.000
30	20	100	5.000
			16.666

$$\therefore \chi^2_{cal} = 16.666$$

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χ^2_{tab} for 1 dof and 5% LOS = 3.84

$$\chi^2_{cal} > \chi^2_{tab}$$

\therefore we reject the Null Hypothesis.

\therefore There is ~~Nature of work does not~~

\therefore Nature of work depends on the gender of the workers.