

2) To study the performance of three detergents and three different water temperature, the following 'whiteness' reading were obtained with designed equipment

Water temp.	Detergent A	Detergent B	Detergent C
Cold water	57	55	67
Warm water	49	52	68
Hot water	54	46	58

Perform a two way analysis of variance, using 5% l.o.s. (Given $F_{tab} 5\% = 6.94$).

Solution:

Step 1: Null Hypothesis: H_0 : There is no ^{significant} difference among three varieties of detergents and also water temperatures

Step 2: Alternative Hypothesis: H_1 : There is significant difference among three varieties of detergents and also water temperatures.

Table 1 Origin = 50 (subtract 50 from each element)

Water Temp	Detergents			
	A (X_1)	B (X_2)	C (X_3)	
(Y_1) Cold	7	5	17	$\Sigma Y_1 = 29$
(Y_2) Warm	-1	2	18	$\Sigma Y_2 = 19$
(Y_3) Hot	4	-4	8	$\Sigma Y_3 = 8$

Table 9:

X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
7	49	5	25	17	289
-1	1	2	4	18	324
4	16	-4	16	8	64
$\sum X_1$	$\sum X_1^2$	$\sum X_2$	$\sum X_2^2$	$\sum X_3$	$\sum X_3^2$
= 10	= 66	= 3	= 45	= 43	= 677

Step 3:

$$N = n_1 + n_2 + n_3 = 3 + 3 + 3 = 9$$

$$r = 3$$

$$c = 3$$

$N = 9$
$r = 3$
$c = 3$

Step 4: $T = \sum X_1 + \sum X_2 + \sum X_3 = 10 + 3 + 43$

$$T = 56$$

Step 5: Correction factor $CF = \frac{T^2}{N} = \frac{(56)^2}{9}$

$$CF = 348.4$$

Step 6: $SST = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - CF$
 $= 66 + 45 + 677 - 348.4$

$$SST = 439.6$$

Step 7: $SSC = \frac{(\sum X_1)^2}{c_1} + \frac{(\sum X_2)^2}{c_2} + \frac{(\sum X_3)^2}{c_3} - CF$
 $= \frac{(10)^2}{3} + \frac{(3)^2}{3} + \frac{(43)^2}{3} - 348.4$

$$SSC = 304.27$$

Step 8: $SSR = \frac{(\sum Y_1)^2}{r_1} + \frac{(\sum Y_2)^2}{r_2} + \frac{(\sum Y_3)^2}{r_3} - CF$
 $= \frac{29^2}{3} + \frac{19^2}{3} + \frac{8^2}{3} - 348.4 = 73.6$

$$SSR = 73.6$$

Step 9: $SSE = SST - SSC - SSR$
 $= 439.6 - 304.27 - 73.6$

$SSE = 61.73$

Step 10: ANOVA Table

Source of Variations	Sum of Squares	Degrees of Freedom	Mean Sum of Squares $= \frac{304.27}{2}$	Variance Ratio F_{cal}	Table Value F_{tab}
B/w Columns	$C-1 = 2$	SSC $= 304.27$	MSC $= 152.14$	$F_C = \frac{152.14}{15.43}$ $= 9.86$	$F_{C tab}$ (2,4) $= 6.94$
B/w Rows	$r-1 = 2$	SSR $= 73.6$	MSR $= \frac{SSR}{r-1}$ $= \frac{73.6}{2} = 36.8$	$F_R = \frac{36.8}{15.43}$ $= 2.38$	$F_{R tab}$ (2,4) $= 6.94$
B/w Errors	$(C-1)(r-1) = 4$	SSE $= 61.73$	MSE $= 15.43$		

Step 11: Conclusion:

- (i) $F_C > F_{C tab}$, we reject H_0 .
 \therefore There is significant difference ~~to~~ among 3 varieties of detergents.
- (ii) $F_R < F_{R tab}$, we accept H_0 .
 \therefore There is no significant difference among 3 water temperatures.