Method III

Gauss-Jordan Method (To find Inverse of a Matrix).

1) Find the Inverse of the matrix using Gauss-Jordan Method
$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ W.R.T $AX = I$ $\boxed{\text{find } X}$

The augmented Matrix (A, I)

$$\simeq \left(\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{array}\right)$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 1 & 3/2 & 1/2 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{array}\right) \quad R_1 \to R_1/2$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 1 & 3/2 & 1/2 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & 2 & 7/2 & -1/2 & 0 & 1 \end{array}\right) \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 1 & 3/2 & 1/2 & 0 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 2 & 7/2 & -1/2 & 0 & 1 \end{array}\right) R_2 \to \frac{R_2}{-1}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 0 & -1/2 & -1/2 & +1 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & -1/2 & -5/2 & 2 & 1 \end{array}\right) \begin{array}{l} R_1 \to R_1 - R_2 \quad \frac{3}{2} - 2 \\ \\ R_3 \to R_3 - 2R_2 \\ \quad -\frac{1}{2} - 2 \end{array}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 0 & -1/2 & -1/2 & 1 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{array}\right) R_3 \to \frac{R_3}{-1/2}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4/2 & -1 & -1 \\ 0 & 1 & 0 & -9 & 7 & 4 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{array}\right) \begin{array}{l} R_1 \to R_1 + \frac{1}{2}R_3 \quad -\frac{1}{2}+\frac{5}{2} \\ \qquad 1 - \frac{4}{2} \\ R_2 \to R_2 - 2R_3 \\ \qquad -1+8 \end{array}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -9 & 7 & 4 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{array}\right)$$

Now $A \simeq$ Identity Matrix.

$\therefore A^{-1}$ is $\begin{pmatrix} 2 & -1 & -1 \\ -9 & 7 & 4 \\ 5 & -4 & -2 \end{pmatrix}$ //

2) Find the inverse of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ Using Gauss - Jordan Method.

Solution:

W.K.T $AX = I$

Let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

The augmented Matrix (A, I)

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right)$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right) \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -4 & 1 & +1 & 1 \end{array}\right) \begin{array}{l} R_2 \to R_2/2 \\ R_3 \to R_3 + R_2 \end{array}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right) \begin{array}{l} R_1 \to R_1 - R_2 \\ \\ R_3 \to R_3/4 \end{array}$$

$$\simeq \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & \frac{3}{2} \\ 0 & 1 & 0 & -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right) \begin{array}{l} R_1 \to R_1 - 6R_3 \\ R_2 \to R_2 + 3R_3 \end{array}$$

$\frac{3}{2} + \frac{6}{\ldots}$ $\frac{12}{\ldots}$ $-\frac{1}{2} + \ldots$ $-\frac{1}{2} - \ldots$ $\frac{1}{2} - \frac{3}{\ldots}$

Now $A \simeq$ Identity Matrix

$\therefore$ Inverse of A is $A^{-1} = \begin{bmatrix} 3 & 1 & \frac{3}{2} \\ -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$

HW: 1) Find inverse of $\begin{pmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{pmatrix}$ Ans: $\frac{1}{56}\begin{pmatrix} 12 & 4 & 6 \\ 1 & 5 & -3 \\ 5 & -3 & -1 \end{pmatrix}$

2) Find inverse of $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$ Ans: $\begin{pmatrix} 5/8 & 1/8 & 0 \\ 7/16 & -7/48 & -1/6 \\ -3/8 & 1/8 & 0 \end{pmatrix}$