Gauss-Elimination Method: (Back Substitution Method)

1) Solve the following system by Gaussian Elimination Method

$$x_1 - x_2 + x_3 = 1$$
$$-3x_1 + 2x_2 - 3x_3 = -6$$
$$2x_1 - 5x_2 + 4x_3 = 5$$

Solution

The Augmented matrix $(A|B) \simeq$

$$\simeq \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{array}\right)$$

$-3 \rightarrow$ pivot element (largest absolute Value)

$$\simeq \left(\begin{array}{ccc|c} -3 & 2 & -3 & -6 \\ 1 & -1 & 1 & 1 \\ 2 & -5 & 4 & 5 \end{array}\right) \quad \begin{array}{l} R_1 \rightarrow R_2 \\ R_2 \rightarrow R_1 \end{array}$$

Bring pivot element to top left corner

$$\simeq \left(\begin{array}{ccc|c} 1 & -2/3 & 1 & 2 \\ 1 & -1 & 1 & 1 \\ 2 & -5 & 4 & 5 \end{array}\right) \quad R_1 \rightarrow \frac{R_1}{-3}$$

Make the top left corner element as 1

$$\simeq \left(\begin{array}{ccc|c} 1 & -2/3 & 1 & 2 \\ 0 & -1/3 & 0 & -1 \\ 0 & -11/3 & 2 & 1 \end{array}\right) \quad \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{array}$$

Make 1st column elements zero

$$\simeq \left(\begin{array}{ccc|c} 1 & -2/3 & 1 & 2 \\ 0 & -11/3 & 2 & 1 \\ 0 & -1/3 & 0 & -1 \end{array}\right) \quad \begin{array}{l} R_2 \rightarrow R_3 \\ R_3 \rightarrow R_2 \end{array}$$

$-11/3 \rightarrow$ pivot element

$$\simeq \left(\begin{array}{ccc|c} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & -1/3 & 0 & -1 \end{array}\right) \quad R_2 \rightarrow \frac{R_2}{(-11/3)}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & -2/11 & -12/11 \end{array}\right) R_3 \to R_3 + \frac{1}{3}R_2$$

$-1 - \cancel{\frac{3}{33}}$

$\frac{-11-3}{11}$

$-\frac{13}{11}$

$$\sim \left(\begin{array}{ccc|c} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & 1 & 6 \end{array}\right) R_3 \to \frac{R_3}{(-2/11)}$$

$-2/11$ pivot element

Upper triangular Matrix.

Back substitution Method,

$$\boxed{z = 6}$$

$$1y - \frac{6}{11} z = -\frac{3}{11}$$

$$y - \frac{6}{11}(6) = -\frac{3}{11}$$

$$y = -\frac{3}{11} + \frac{36}{11}$$

$$y = \frac{33}{11}$$

$$\boxed{y = 3}$$

$$x - 2/3\, y + z = 2$$

$$x - \frac{2}{3}(3) + 6 = 2$$

$$x - 2 + 6 = 2$$

$$x = 2 - 4$$

$$\boxed{x = -2}$$

$$\therefore \boxed{x_1 = -2} \boxed{x_2 = 3} \boxed{x_3 = 6}.$$

Check:

(1) $\Rightarrow -2 - 3 + 6$

$= -5 + 6 = 1$ ✓

2) Solve by Gauss-Elimination Method.

$$3x + y - z = 3$$
$$2x - 8y + z = -5$$
$$x - 2y + 9z = 8$$

Solution: The Augmented Matrix (A/B)

$$\simeq \left(\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array}\right) \quad 3 \rightarrow \text{pivot element}$$

$$\simeq \left(\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 1 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array}\right) \quad R_1 \rightarrow \frac{R_1}{3}$$

$$\simeq \left(\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 1 \\ 0 & -26/3 & 5/3 & -7 \\ 0 & -7/3 & 28/3 & 7 \end{array}\right) \quad \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$-8-\frac{2}{3}$, $1+\frac{2}{3}$, $-5+2$, $-2-\frac{1}{3}$, $9+\frac{1}{3}$, $8-1$

$$\simeq \left(\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 1 \\ 0 & 1 & -5/26 & 21/26 \\ 0 & -7/3 & 28/3 & 7 \end{array}\right) \quad R_2 \rightarrow \frac{R_2}{(-26/3)}$$

$-26/3$ is pivot element

$$\simeq \left(\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 1 \\ 0 & 1 & -5/26 & 21/26 \\ 0 & 0 & 231/26 & \frac{231}{26} \end{array}\right) \quad R_3 \rightarrow R_3 + \frac{7}{3} R_2$$

$\frac{28}{3} - \frac{7}{3}\frac{5}{26}$

$\frac{1}{3}\left(\frac{28 \times 26 - 35}{26}\right)$

$7 + \frac{7}{3}\left(\frac{21}{26}\right)$

$$\simeq \left(\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 1 \\ 0 & 1 & -5/26 & 21/26 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad R_3 \rightarrow \frac{R_3}{\left(\frac{231}{26}\right)}$$

$\frac{693}{3} = 231$

281, 26, 168, 56, 728

Using Back Substitution Method

$$\boxed{z = 1}$$

$$y - \frac{5}{26}z = \frac{21}{26}$$

$$y = \frac{21}{26} + \frac{5}{26}$$

$$y = \frac{26}{26}$$

$$\boxed{y = 1}$$

$$x + \frac{1}{3}y - \frac{1}{3}z = 1$$

$$x = 1 + \frac{1}{3} - \frac{1}{3}$$

$$\boxed{x = 1}$$

Solution

$$\boxed{\begin{array}{l} x = 1 \\ y = 1 \\ z = 1 \end{array}}$$