Gauss-Jacobi's Method / Jacobi's Method (Iterative Method)

1) Solve using Gauss-Jacobi Method.

$10x - 5y - 2z = 3$

$4x - 10y + 3z = -3$

$x + 6y + 10z = -3$

Solution:

$10x = 3 + 5y + 2z$

$x = \frac{1}{10}(3 + 5y + 2z)$ — ①

$-10y = 3 - 4x - 3z$

$y = \frac{1}{-10}(3 - 4x - 3z)$ — ②

$10z = -3 - x - 6y$

$z = \frac{1}{10}(-3 - x - 6y)$ — ③

check:

$|10| > |-5| + |-2|$

$|-10| > |4| + |3|$

$|10| > |1| + |6|$

since diagonals are dominant, this method is applied

The gn system can be rewritten as

$$x = \frac{1}{10}(3 + 5y + 2z)$$

$$y = \frac{-1}{10}(-3 - 4x - 3z)$$

$$z = \frac{1}{10}(-3 - x - 6y)$$

Iteration - 1 Initial Values $x_0 = y_0 = z_0 = 0$

$$x_1 = \frac{1}{10}(3 + 0 + 0) = 0.3$$

$$y_1 = \frac{-1}{10}(3 - 0 - 0) = 0.3$$

$$z_1 = \frac{1}{10}(-3 - 0 - 0) = -0.3$$

Table:

Iteration	$x = \frac{1}{10}(3+5y+2z)$	$y = \frac{1}{10}(3+4x+3z)$	$z = \frac{1}{10}(-3+4$
1	$x_1 = 0.3$	$y_1 = 0.3$	$z_1 = -0.3$
2	$x_2 = \frac{1}{10}(3+5(0.3)+2(-0.3)) = 0.39$	$y_2 = \frac{1}{10}(3+4(0.3)+3(-0.3)) = 0.33$	$z_2 = \frac{1}{10}(-3.0 - 6(0.3) = -0.51$
3	$x_3 = 0.363$	$y_3 = 0.303$	$z_3 = -0.531$
4	$x_4 = 0.3441$	$y_4 = 0.2841$	$z_4 = -0.5181$
5	$x_5 = 0.3384$	$y_5 = 0.2822$	$z_5 = -0.5048$
6	$x_6 = 0.3401$	$y_6 = 0.2839$	$z_6 = -0.5031$
7	$x_7 = 0.3413$	$y_7 = 0.2851$	$z_7 = -0.5043$
8	$x_8 = 0.3416$	$y_8 = 0.2852$	$z_8 = -0.5051$
9	$x_9 = 0.3415$	$y_9 = 0.2851$	$z_9 = -0.5052$
10	$x_{10} = 0.3415$	$y_{10} = 0.2851$	$z_{10} = -0.5052$

Since 9th and 10th Iterations are equal,

$x \simeq 0.3415 \mid y \simeq 0.2851 \mid z \simeq -0.5052$

2) Solve using Jacobi's Iteration Method.

$30x - 2y + 3z = 75$

$x + 17y - 2z = 48$

$x + y + 9z = 15$

Solution: $|30| > |-2| + |3|$

$|17| > |1| + |-2|$

$|9| > |1| + |1|$

Since the diagonal elements are dominant, we can apply Jacobi Method.

The given can be written as

$30x = 75 + 2y - 3z \Rightarrow x = \frac{1}{30}(75 + 2y - 3z)$

$17y = 48 - x + 2z \Rightarrow y = \frac{1}{17}(48 - x + 2z)$

$9z = 15 - x - y \Rightarrow z = \frac{1}{9}(15 - x - y)$

Take Initial Values $x_0 = y_0 = z_0 = 0$

Table:

Iteration	$x = \frac{1}{30}(75 + 2y - 3z)$	$y = \frac{1}{17}(48 - x + 2z)$	$z = \frac{1}{9}(15 - x - y)$
1	$x_1 = 2.5$	$y_1 = 2.8235$	$z_1 = 1.6667$
2	$x_2 = 2.5217$	$y_2 = 2.8725$	$z_2 = 1.0751$
3	$x_3 = 2.5839$	$y_3 = 2.8016$	$z_3 = 1.0673$
4	$x_4 = 2.5800$	$y_4 = 2.7971$	$z_4 = 1.0682$
5	$x_5 = 2.5796$	$y_5 = 2.7974$	$z_5 = 1.0692$
6	$x_6 = 2.57965$	$y_6 = 2.7975$	$z_6 = 1.0692$
7	$x_7 = 2.5795$	$y_7 = 2.7975$	$z_7 = 1.0692$

Since 6th & 7th Iterations are equal

$x \simeq 2.5795 \quad y \simeq 2.7975 \quad z \simeq 1.0692.$

HW: Jacobi's Method $20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

4th approximation — $x = 1 ; y = -1 ; z = 1$

$x = 1.00039 \,|\, y = -1.00003 \,|$

$z = 0.99965$