Gauss-Seidal Iterative Method

1) Solve the system of equations using Gauss-Seidel Iteration Method.

$$x+y+54z=110$$
$$27x+6y-z=85$$
$$6x+15y+2z=72$$

Solution:

The given system is

Interchanging the equations,

$$27x+6y-z=85$$
$$6x+15y+2z=72$$
$$x+y+54z=110$$

$|27| > |6|+|-1|$
$|15| > |6|+|2|$
$|54| > |1|+|1|$

The diagonal elements are dominant, we can apply Seidal Method.

$$27x = 85-6y+z \Rightarrow x = \frac{1}{27}(85-6y+z)$$

$$15y = 72-6x-2z \Rightarrow y = \frac{1}{15}(72-6x-2z)$$

$$54z = 110-x-y \Rightarrow z = \frac{1}{54}(110-x-y)$$

Initial Values: $y_0 = z_0 = 0$

Iteration	$x=\frac{1}{27}(85-6y+z)$	$y=\frac{1}{15}(72-6x-2z)$	$z=\frac{1}{54}(110-x-y)$
1	$x_1 = 3.148$	$y_1 = 3.5408$	$z_1 = 1.913$
	2.4322	3.572	1.92585
2	$x_2 = 2.715$	$y_2 = 3.458$	$z_2 = 1.923$
	2.42569	3.5729	1.92595
3	$x_3 = 2.735$	$y_3 = 3.449$	$z_3 = 1.923$
	2.42550	3.573	1.92595
4	$x_4 = 2.738$	$y_4 = 3.4484$	$z_4 = 1.9224$
	2.42547	3.5730	1.92595
5	$x_5 = 2.738$	$y_5 = 3.4484$	$z_5 = 1.9224$
	2.42547	3.5730	1.92595

since 4th & 5th Iterations are equal,

$x \simeq 2.738$ (2.42547)

$y \simeq 3.4484$ (3.5730)

$z \simeq 1.9224$ (1.92595)

2) Solve Using Gauss-Seidal Method.

$4x+2y+z=14 \quad | \quad x+5y-z=10 \quad | \quad x+y+8z=20$

Solution: The gn system is $4x+2y+z=14$, $x+5y-z=10$, $x+y+8z=20$

$|4| > |2|+|1|$

$|5| > |1|+|-1|$

$|8| > |1|+|1|$

The diagonal elements are dominant, we can apply Seidal Method.

$4x = 14-2y-z \Rightarrow x = \frac{1}{4}(14-2y-z)$

$5y = 10-x+z \Rightarrow y = \frac{1}{5}(10-x+z)$

$8z = 20-x-y \Rightarrow z = \frac{1}{8}(20-x-y)$

Initial Values: $y_0 = z_0 = 0$

Iteration	$x=\frac{1}{4}(14-2y-z)$	$y=\frac{1}{5}(10-x+z)$	$z=\frac{1}{8}(20-x-y)$
1	$x_1=3.5$	$y_1=1.3$	$z_1=1.9$
2	$x_2=2.375$	$y_2=1.905$	$z_2=1.965$
3	$x_3=2.056$	$y_3=1.982$	$z_3=1.995$
4	$x_4=2.010$	$y_4=1.997$	$z_4=1.999$
5	$x_5=2.001$	$y_5=1.999$	$z_5=2$
6	$x_6=2.001$	$y_6=1.999$	$z_6=2$

Since 5th and 6th Iterations are equal,

$x \simeq 2.001 \quad y \simeq 1.999 \quad z \simeq 2.$

HW: $8x-y+z=18$, $2x+5y-2z=3$, $x+y-3z=-6$ (Gauss-Seidal Method).

$x = 1.9999$, $y = 0.9999$, $z = 2.9999$ 6th Iteration.