Student t-test (SD is not given directly)

1) A random sample of 10 boys had the following IQ's : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of a population mean IQ of 100?

Find a reasonable range in which most of the mean IQ values of samples of 10 boys lie.

Solution:

Given: $n = 10 < 30$ [small sample]

$\mu = 100$

x	$x - 97.2$ $x - \bar{x}$	$(x - \bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
$\Sigma x = 972$		1833.60

Sample Mean $\bar{x} = \frac{\Sigma x}{n} = \frac{972}{10} = 97.2$

$$s^2 = \frac{1}{n-1}\Sigma(x_i - \bar{x})^2 = \frac{1}{9}(1833.60)$$

$$s^2 = 203.73$$

$$s = \sqrt{203.73}$$

$$\boxed{s = 14.27}$$

We use student t-test.

1) Null Hypothesis: $H_0 : \mu = 100$

population mean IQ is 100.

2) Alternative Hypothesis $H_1 : \mu \neq 100$ [Two-tailed]

*) Test Statistic $t_{cal} = \frac{\bar{x} - \mu}{(s/\sqrt{n})} = \frac{97.2 - 100}{(14.27/\sqrt{10})} = \frac{-2.8}{4.514}$

$$t_{cal} = -0.62$$

$$\boxed{|t_{cal}| = 0.62}$$

*) Degrees of freedom: $\nu = n-1 = 10-1 = 9$

LoS = 5%.

*) t_{tab} at 5% with 9 d.o.f; $\boxed{t_{tab} = 2.26}$

$t_{cal} < t_{tab}$, ∴ H_0 is accepted.

∴ The data support that the population mean IQ is 100.

*) Confidence limit:

$$\mu = x \pm t_{0.05}\frac{s}{\sqrt{n}}$$

$$= 97.2 \pm \left[2.26 \times \left(\frac{14.27}{\sqrt{10}}\right)\right]$$

$$= 97.2 \pm 10.20$$

$$= 107.40 \text{ and } 87.00.$$

∴ The reasonable range is [87.00, 107.40].

2) The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level assuming that for 9 degrees of freedom $P(t > 1.83) = 0.05$

Solution:

Given: $n = 10 < 30$ [small sample]

$\mu = 64$

x	$x - \bar{x}$	$(x - \bar{x})^2$
70	4	16
67	1	1
62	−4	16
68	2	4
61	−5	25
68	2	4
70	4	16
64	−2	4
64	−2	4
66	0	0

Σx 660 $\qquad \Sigma(x - \bar{x})^2 = 90.$

Mean, $\bar{x} = \frac{\Sigma x}{n} = \frac{660}{10} = 66.$

$$s^2 = \frac{1}{n-1} \Sigma (x - \bar{x})^2 = \frac{1}{9}(90) = 10$$

$s^2 = 10 \qquad s = \sqrt{10}$

sample SD $= s = 3.16$ //

We use student-t test.

*) Null Hypothesis: $H_0 : \mu = 64$

The average height is not more than 64.

*) Alternative Hypothesis: $H_1 : \mu > 64$ (one-tailed test)

*) Test statistic: $t_{cal} = \dfrac{\bar{x} - \mu}{(s/\sqrt{n})} = \dfrac{66 - 64}{(3.16/\sqrt{10})} = 2.$

$$\boxed{t_{cal} = 2}.$$

*) Degrees of freedom: $\nu = n-1 = 10-1 = 9$

*) t_{tab} at 5% with 9 d.o.f ~~[illegible]~~

is $t_{tab} = 1.833$ (Given).

$t_{cal} > t_{tab}$, $\therefore$ H_0 is rejected.

$\therefore$ The average height is greater than 64 inches.