Test of Significance for Difference of Means :

<u>Test statistic</u> : $Z_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

If $\sigma_1 = \sigma_2 = \sigma$, $Z_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

<u>formula</u>: $\sigma^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$

<u>Problem 1</u>: The means of 2 large samples 1000 and 2000 members are ~~a~~ 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D. 2.5 inches.

<u>Solution</u>:

<u>Given</u>: $n_1 = 1000$ $\quad n_2 = 2000$

$\bar{x}_1 = 67.5 \quad \bar{x}_2 = 68$

$\sigma = 2.5$ inches.

Null Hypothesis : $H_0 : \mu_1 = \mu_2$

Two samples drawn from the same population of S.D. 2.5 inches.

Alternative Hypothesis : $H_1 : \mu_1 \neq \mu_2$ (Two tailed Test)

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{67.5 - 68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}} = \frac{-0.5}{0.0968}$$

$$= -5.16.$$

$|Z_{cal}| = 5.16$

$Z_{tab} = 1.96$

Here, $|Z_{cal}| > Z_{tab}$

$\therefore$ We reject H_0 at 5% LoS.

$\therefore$ The samples are not drawn from the same population of S.D. 2.5 inches

2). A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that Americans are the average taller than the English men?

~~$\sigma_1^2 = 40.96$~~ | ~~$\sigma_2^2 =$~~

Gn: $n_1 = 6400$ $\quad \bar{x}_1 = 170$ $\quad \sigma_1 = 6.4$

$n_2 = 1600$ $\quad \bar{x}_2 = 172$ $\quad \sigma_2 = 6.3$.

Null Hypothesis: $H_0 : \mu_1 = \mu_2$

Both means do not differ significantly.

Alternative Hypothesis: $H_1 ; \mu_1 > \mu_2$

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{170 - 172}{\sqrt{\frac{40.96}{6400} + \frac{39.69}{1600}}} = \frac{-2}{\sqrt{0.025}}$$

$|Z_{cal}| = 12.658$.

$Z_{tab} = 2.33$

$Z_{cal} > Z_{tab}$

∴ we reject null hypothesis H_0.

∴ Americans are taller than the Englishmen.

Nature of Test	Level of Significance.		
	1 %	5 %	10 %
Two-tailed test	2.58	1.96	1.645
One-tailed test	2.33	1.645	1.28 (right)
	−2.33	−1.645	−1.28 (left)

3.) The average hourly wages of a sample of 100 workers in plant A was ₹ 2.56 with SD 1.08. The average wages of a sample of 200 workers in plant B was ₹. 2.87 with SD 1.28 on average. Assume that hourly wages paid by plant B ~~of~~ ~~this~~ there was higher than plant A.

<u>Solution:</u>

$n_1 = 150$ $\quad s_1 = 1.08$ $\quad \bar{x}_1 = 2.56$

$n_2 = 200$ $\quad s_2 = 1.28$ $\quad \bar{x}_2 = 2.87$

Null Hypothesis: $H_0 : \mu_1 = \mu_2$

i.e., they do not differ.

Alternative Hypothesis: $H_1 : \mu_1 < \mu_2$ (left tailed)

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} = \frac{\overset{174.96}{(150)(1.08)^2} + \overset{327.68}{(200)(1.28)^2}}{150 + 200}$$

$$= \frac{502.64}{350} = 1.436$$

$\boxed{\sigma = 1.1983}$

$$Z_{cal} = \frac{2.56 - 2.87}{\sqrt{1.436}\sqrt{\frac{1}{150} + \frac{1}{200}}} = \frac{-0.31}{(1.1983)(0.1079)}$$

$$= \frac{-0.31}{0.1292}$$

$$= -2.39$$

$$|Z_{cal}| = +2.39$$

Critical value : LOS - 5 %

$$|Z_{tab}| = -1.645 \text{ (left tailed)}$$

$\therefore$ $Z_{cal} > Z_{tab}$

$\therefore$ we reject H_0,

$\therefore$ Hourly wages are paid higher by plant A.