Latin Square Design:

1) An agriculturist wants to test the effects of four different fertilizers A, B, C and D on the yi of paddy. In order to eliminate sources of erro due to variability in self-fertility he used the fertilizers in a Latin square arrangement given below where the numbers indicate yield in quintals per unit area. Perform an analysi of variance to decide whether there is significa difference b/w the fertilizers at 5% LoS.

A 18	D 21	C 23	B 11
D 22	A 20	B 10	C 19
B 15	C 21	D 25	A 17
C 22	B 12	A 15	D 24

Solution:

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Step 1: Null Hypothesis: H_0: There is no significant difference b/w the fertilizers.

Step 2: Alternate Hypothesis: H_1: There is significant difference b/w the fertilizers.

Step 3: Table 1 Set Origin $= x - \left(\frac{max + min}{2}\right) = x - \left(\frac{25+10}{2}\right) = x - 18$

	x_1 1	x_2 2	x_3 3	x_4 4	
Y_1 1	0	3	5	−7	$\Sigma Y_1 = 1$
Y_2 2	4	2	−8	1	$\Sigma Y_2 = -1$
Y_3 3	−3	3	7	−1	$\Sigma Y_3 = 6$
Y_4 4	4	−6	−3	6	$\Sigma Y_4 = 1$

Table - 2:

x_1	x_1^2	x_2	x_2^2	x_3	x_3^2	x_4	x_4^2
0	0	3	9	5	25	-1	49
4	16	2	4	-8	64	1	1
-3	9	3	9	7	49	-1	1
4	16	-6	36	-3	9	6	36
$\Sigma x_1 = 5$	$\Sigma x_1^2 = 41$	$\Sigma x_2 = 2$	$\Sigma x_2^2 = 58$	$\Sigma x_3 = 1$	$\Sigma x_3^2 = 147$	$\Sigma x_4 = -1$	$\Sigma x_4^2 = 87$

Step-4: $T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$

$= 5 + 2 + 1 - 1 = 7$ $\boxed{T = 7}$

Step-3: $N = n_1 + n_2 + n_3 + n_4 = 4 + 4 + 4 + 4 = 16$

$\boxed{r = 4}$ $\boxed{c = 4}$ $\boxed{N = 16}$

Step 5: Correction factor $CF = T^2/N = 7^2/16 = \frac{49}{16}$.

$\boxed{CF = 3.0625}$

Step 6: $SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - CF$

$= 41 + 58 + 147 + 87 - 3.0625$

$= 333 - 3.0625$

$\boxed{SST = 329.94}$

Step 7: $SSC = \frac{(\Sigma x_1)^2}{c_1} + \frac{(\Sigma x_2)^2}{c_2} + \frac{(\Sigma x_3)^2}{c_3} + \frac{(\Sigma x_4)^2}{c_4} - CF$

$= \frac{(5)^2}{4} + \frac{2^2}{4} + \frac{1^2}{4} + \frac{(-1)^2}{4} - 3.0625$

$= \frac{25}{4} + \frac{4}{4} + \frac{1}{4} + \frac{1}{4} - 3.0625$

$\boxed{SSC = 4.6875}$

Step 8: $SSR = \frac{(\Sigma Y_1)^2}{r_1} + \frac{(\Sigma Y_2)^2}{r_2} + \frac{(\Sigma Y_3)^2}{r_3} + \frac{(\Sigma Y_4)^2}{r_4} - CF$

$= \frac{1^2}{4} + \frac{(-1)^2}{4} + \frac{6^2}{4} + \frac{1^2}{4} - 3.0625 = 6.6875$

Step 9: Table 3

To find TSS

(z_1) A	0	2	−3	−1	$\Sigma z_1 = -2$
(z_2) B	−3	−6	−8	−7	$\Sigma z_2 = -24$
(z_3) C	4	3	5	1	$\Sigma z_3 = 13$
(z_4) D	4	3	7	6	$\Sigma z_4 = 20$

$$TSS = \frac{(\Sigma z_1)^2}{z_1} + \frac{(\Sigma z_2)^2}{z_2} + \frac{(\Sigma z_3)^2}{z_3} + \frac{(\Sigma z_4)^2}{z_4} - C$$

$$= \frac{(-2)^2}{4} + \frac{(-24)^2}{4} + \frac{(13)^2}{4} + \frac{(20)^2}{4} - 3.06$$

$$= \frac{4}{4} + \frac{24 \times 24}{4} + \frac{169}{4} + \frac{400}{4} - 3.0625$$

$$= 287.25 - 3.0625$$

$$\boxed{TSS = 284.1875}$$

Step 10: $SSE = SST - SSC - SSR - TSS$

$$= 329.94 - 4.6875 - 6.6875 - 284.1875$$

$$\boxed{SSE = 34.375}$$

Step 11 ANOVA Table

Source of Variation	Sum of Squares	Deg of freedom	Mean Sum of Squares	Variance Ratio	Table Value
B/w columns	SSC = 4.6875	c−1 = 3	$MSC = \frac{SSC}{c-1} = 1.5625$	$F_C = \frac{5.7291}{1.5625} = 3.6666$	F_C tab (6,3) = 9.94
B/w rows	SSR = 6.6875	r−1 = 3	$MSR = \frac{SSR}{r-1} = 2.2291$	$F_R = \frac{5.7291}{2.2291} = 2.5701$	F_R tab (6,3) = 9.
B/w treatments	TSS = 284.1875	z−1 = 3	$MST = \frac{TSS}{z-1} = 94.7291$	$F_T = \frac{94.7291}{5.7291} = 16.53$	F_T tab (3,6) = 4.76
B/w errors	SSE = 34.375	(c−1)(r−2) = 3.2 = 6	$MSE = \frac{SSE}{(c-1)(r-2)} = 5.7291$		

Step 12: Conclusion

(i) $f_{c\,cal} < f_{c\,tab}$, We accept H_0

(ii) $f_{R\,cal} < f_{R\,tab}$, We accept H_0

(iii) $f_{T\,cal} > f_{T\,tab}$, We reject H_0.

ie., We accept Null Hypothesis.

i.e., There is significant difference b/w the fertilizers.

2) Five varieties of paddy A, B, C, D and E are tried. The plan, the varieties shown in each plot and yields obtained in kg are given in the following table.

B 95	E 85	C 139	A 117	D 97
E 90	D 89	B 75	C 146	A 87
C 116	A 95	D 92	B 89	E 74
A 85	C 130	E 90	D 81	B 77
D 87	B 65	A 99	E 89	C 93.

Test whether there is a significant difference b/w rows and columns at 5% LoS.