GENERAL SOLUTION OF THE TRANSMISSION LINE

→ Transmission line general solution is used to find the voltage and current at any point on a line.

The notations used in transmission lines are

R – Series resistance, ohms per unit length of the line

L – Series inductance, henrys per unit length of the line

G – shunt leakage conductance between conductors, mhos per unit length of the line

C – capacitance between conductors, farads per unit length of the line

ωL – Series reactance, ohms per unit length of the line

$Z = R + j\omega L$ → series impedance, ohms per unit length of the line

ωC – shunt susceptance, mhos per unit length of the line.

$Y = G + j\omega C$ – shunt admittance, mhos per unit length of the line

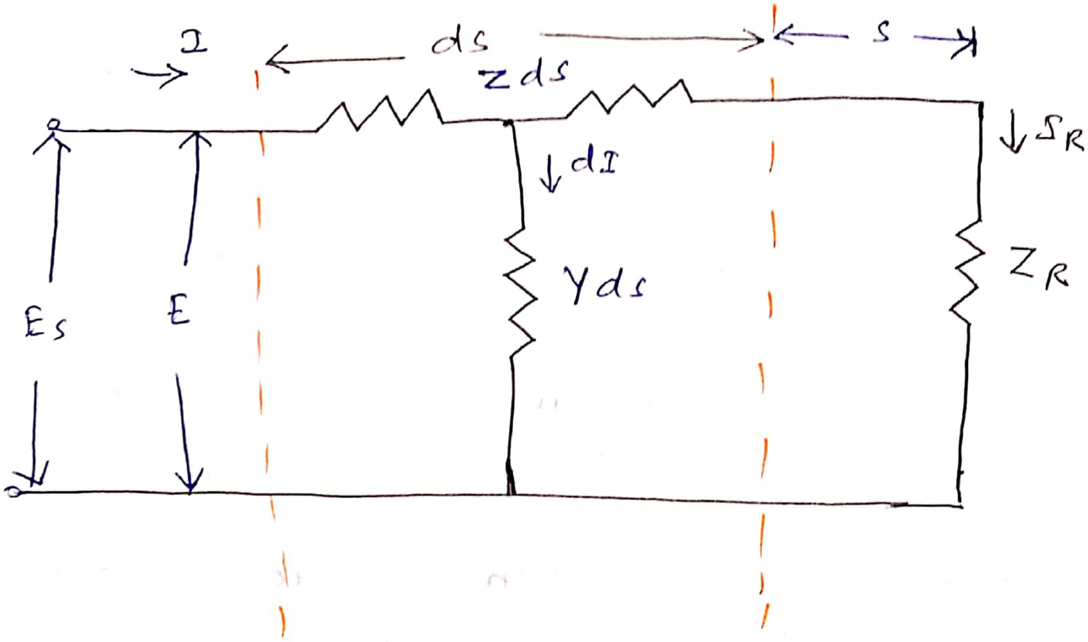
s – distance to the point of observation, measured from the receiving end of the line

I – current in the line at any point

E - voltage between conductors at any point

l - length of the line.

General solution



(Fig) A long line, with the elements of one of the small section shown.

* Figure illustrates a line may be considered as made up of cascaded T sections, one of which is shown.

* This elemental section is of length ds and carries a current 'I'.

* The series line impedance being z ohms per unit, the series impedance of the given elemental section is zds ohms.

The voltage drop in the length ds is

$$dE = I z ds \quad \rightarrow (1)$$

$$\frac{dE}{ds} = IZ \rightarrow (2)$$

The shunt admittance per unit length of line is Y mhos, so that the admittance of the element of line is Yds mhos.

The current dI that flows across the line (or) from one conductor to the other is,

$$dI = EYds \rightarrow (3)$$

$$\frac{dI}{ds} = EY \rightarrow (4)$$

Eqns (2) & (4) are differentiated w.r.t. 's'

eq (2) $\rightarrow$ $$\frac{d^2E}{ds^2} = Z\frac{dI}{ds} \rightarrow (5)$$

subs eq (4) in eq (5)

$$\frac{d^2E}{ds^2} = ZEY \rightarrow (6)$$

eq (4) $\rightarrow$ $$\frac{d^2I}{ds^2} = Y\frac{dE}{ds} \rightarrow (7)$$

Subs eq (2) in eq (7)

$$\frac{d^2I}{ds^2} = YIZ \rightarrow (8)$$

Eqns (6) & (8) are the differential eqns. of the transmission line.

Interms of operator 'm' eqn. (6) can be written as,

$$\left(\frac{d^2}{ds^2} \to m^2\right)$$

$$(m^2 - zy) E = 0$$

$$m^2 - zy = 0$$

$$m^2 = zy$$

$$m = \pm\sqrt{zy}$$

The solution is,

$$\therefore \quad E = Ae^{\sqrt{zy}\, s} + Be^{-\sqrt{zy}\, s} \to (9)$$

|||ly from eqn. (8)

$$I = Ce^{\sqrt{zy}\, s} + De^{-\sqrt{zy}\, s} \to (10)$$

where A, B, C & D are arbitrary constants of integration.

Since distance 's' is measured from the receiving end, at

$$\boxed{s = 0, \ I = I_R \ \& \ E = E_R}$$

subs in eqns (9) & (10)

$$E = A + B \to (11)$$

$$I = C + D \to (12)$$

differentiating eqn (9) & (10) w.r.t. 's'

$$\frac{dE}{ds} = Ae^{\sqrt{zy}\, s} \cdot \sqrt{zy} + Be^{-\sqrt{zy}\, s}(-\sqrt{zy}) \to (13)$$

$$\frac{dI}{ds} = Ce^{\sqrt{zy}\, s} \cdot \sqrt{zy} + De^{-\sqrt{zy}\, s}(-\sqrt{zy}) \to (14)$$

Subs eqns (2) & (4) in eqns (13) & (14)

$$I z = A\sqrt{zy}\, e^{\sqrt{zy}\, s} - B\sqrt{zy}\, e^{-\sqrt{zy}\, s}$$

$$\therefore\ I = A\sqrt{\frac{Y}{Z}}\, e^{\sqrt{zy}\, s} - B\sqrt{\frac{Y}{Z}}\, e^{-\sqrt{zy}\, s} \rightarrow (15)$$

From (14)

$$EY = C\sqrt{zy}\, e^{\sqrt{zy}\, s} - D\sqrt{zy}\, e^{-\sqrt{zy}\, s}$$

$$\therefore\ E = C\sqrt{\frac{Z}{Y}}\, e^{\sqrt{zy}\, s} - D\sqrt{\frac{Z}{Y}}\, e^{-\sqrt{zy}\, s} \rightarrow (16)$$

subs the condition $s=0$, $E = E_R$ & $I = I_R$ in equ (15) & (16)

$$I_R = A\sqrt{Y/Z} - B\sqrt{Y/Z} \rightarrow (17)$$

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \rightarrow (18)$$

eq (12) $\times \sqrt{\frac{Z}{Y}}$

$$I_R\sqrt{\frac{Z}{Y}} = C\sqrt{\frac{Z}{Y}} + D\sqrt{\frac{Z}{Y}}$$

$$E_R + I_R\sqrt{\frac{Z}{Y}} = 2C\sqrt{\frac{Z}{Y}}$$

$$\therefore\ C = \frac{E_R}{2\sqrt{\frac{Z}{Y}}} + \frac{I_R}{\cancel{2\sqrt{\frac{Z}{Y}}}}\cancel{\sqrt{\frac{Z}{Y}}}$$

$$= \frac{E_R}{2}\sqrt{\frac{Y}{Z}} + \frac{I_R}{2} \qquad \rightarrow \left(Z_0 = \sqrt{\frac{Z}{Y}}\right)$$

$I_R Z_R$

$$C = \frac{E_R}{2}\sqrt{\frac{Y}{Z}} + \frac{I_R}{2} = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)$$

IIIly other parameters can be found

$$A = \frac{E_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)$$

$$B = \frac{E_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)$$

$$C = \frac{I_R}{2} + \frac{E_R}{2}\sqrt{\frac{Y}{Z}} = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)$$

$$D = \frac{I_R}{2} - \frac{E_R}{2}\sqrt{\frac{Y}{Z}} = \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)$$

subs the values in eq 9 & 10.

$$E = A e^{\sqrt{ZY}s} + B e^{-\sqrt{ZY}s} \rightarrow 9$$

$$E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}s}$$

$$= \frac{E_R}{2}e^{\sqrt{ZY}s} + \frac{Z_0 E_R}{2Z_R}e^{\sqrt{ZY}s} + \frac{E_R}{2}e^{-\sqrt{ZY}s} - \frac{E_R Z_0}{2Z_R}e^{-\sqrt{ZY}s}$$

$$= \frac{E_R}{2}e^{\sqrt{ZY}s} + \frac{Z_0 I_R \cancel{Z_R}}{2\cancel{Z_R}}e^{\sqrt{ZY}s} + \frac{E_R}{2}e^{-\sqrt{ZY}s} - \frac{I_R \cancel{Z_R} Z_0}{2\cancel{Z_R}}e^{-\sqrt{ZY}s}$$

$$= E_R\left(\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}}{2}\right) + I_R Z_0\left(\frac{e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}}{2}\right)$$

$$\boxed{E = E_R \cosh\sqrt{ZY}s + I_R Z_0 \sinh\sqrt{ZY}s} \rightarrow 19$$

$$\text{IIIly} \quad \boxed{I = I_R \cosh\sqrt{ZY}s + \frac{E_R}{Z_0}\sinh\sqrt{ZY}s} \rightarrow 20$$

Eqns 19 & 20 are standard form of voltage and current at any point on a line.

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Another form

Eqn. (9)

$$E = A e^{\sqrt{ZY}s} + B e^{-\sqrt{ZY}s}$$

Subs the values of A & B

$$E = \frac{E_R}{2}\left(1+\frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \frac{E_R}{2}\left(1-\frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}s}$$

$$= \frac{E_R}{2}\left[e^{\sqrt{ZY}s} + \frac{Z_0}{Z_R}e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s} - \frac{Z_0}{Z_R}e^{-\sqrt{ZY}s}\right]$$

$$E = \frac{E_R}{2}\left[\left(1+\frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \left(1-\frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}s}\right]$$

$$E = \frac{E_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \left(\frac{Z_R-Z_0}{Z_R}\right)e^{-\sqrt{ZY}s}\right]$$

$$\boxed{E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\sqrt{ZY}s} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}s}\right]} \rightarrow (21)$$

Ily

$$\boxed{I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\sqrt{ZY}s} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}s}\right]} \rightarrow (22)$$

The above eqns are another standard form of eqns. for voltage and current at any point on a transmission line.