

WAVEFORM DISTORTION & DISTORTIONLESS TRANSMISSION LINE

Determination of α & β

$$\text{W.K.T } Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\gamma = \sqrt{ZY}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \rightarrow (1)$$

Squaring both sides,

$$(\alpha + j\beta)^2 = (R + j\omega L)(G + j\omega C)$$

$$\alpha^2 - \beta^2 - j2\alpha\beta = RG + j\omega RC + j\omega LG - \omega^2 LC \rightarrow (2)$$

Equating real terms,

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \rightarrow (3)$$

Equating imaginaries & squaring

$$4\alpha^2\beta^2 = \omega^2(LG + RC)^2 \rightarrow (4)$$

Subs eq (3) in eq (4) & solving,

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2(LG + RC)^2}}{2}}$$

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2(LG + RC)^2}}{2}}$$

WAVEFORM DISTORTION

- * Signals transmitted over lines are normally complex and consists of many frequency components.
- * For ideal transmission, the waveform at the receiving end must be same as the waveform of the original input signal.
- * This requires that all frequencies have same attenuation and the same delay caused by a finite phase velocity or velocity of propagation.
- * When these conditions are not satisfied distortion exists. The distortions occurring in the transmission line are called waveform distortion or ~~delay~~ line distortion.

Two types

- (i) Frequency distortion
- (ii) Phase or delay distortion.

(i) Frequency distortion

- * When a signal having many frequency components are transmitted along the line, all the frequencies will not have equal attenuation.
- * Ex: Voice signal is a complex waveform consists of many frequencies, will not have all frequencies transmitted with equal attenuation.
- * And the received waveform will not be identical

with the input waveform at the sending end.

- * This variation is known as frequency distortion.
- * Frequency distortion is reduced in the transmission line of high quality radio broadcast programme over wire lines by the use of equalizers at the line terminals.
- * These circuits are networks whose frequency and phase characteristics are adjusted to be inverse to those of the lines, resulting in an over-all uniform frequency response over the desired frequency band.

(ii) Phase (or) delay distortion

- * When a signal having many frequency components are transmitted along the line, all the frequencies will not have same time of transmission.
- * Some frequencies being delayed more than the others.
- * So the received end waveform will not be identical with the input waveform at the sending end because some frequency components will be delayed more than those of other frequencies.
- * This type of distortion is called phase or delay distortion.

The distortionless line

If a line is to have neither frequency nor delay distortion, then α and the velocity of propagation

can not be functions of frequency.

α & V \rightarrow independent of frequency
 β \rightarrow dependant on frequency.

Expression for β ,

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + \omega^2(LG + RC)^2}}{2}}$$

The term under the second radical is equated to $(RG + \omega^2 LC)^2$ then the required condition for β is obtained.

$$\begin{aligned} \therefore (RG - \omega^2 LC)^2 + \omega^2(LG + RC)^2 &= (RG + \omega^2 LC)^2 \\ R^2G^2 + \omega^4L^2C^2 - 2RG\omega^2LC + \omega^2L^2G^2 + \omega^2R^2C^2 + 2\omega^2LGRC & \\ &= R^2G^2 + \omega^4L^2C^2 + 2RG\omega^2LC \end{aligned}$$

$$\omega^2(L^2G^2 + R^2C^2 - 2LGRC) = 0$$

$$L^2G^2 + R^2C^2 - 2LGRC = 0$$

$$(LG - RC)^2 = 0$$

$$LG - RC = 0$$

$$LG = RC$$

$$\boxed{\frac{L}{C} = \frac{R}{G}}$$

This is the condition for distortionless transmission line.

Subs $(R_G + \omega^2 LC)^2$ in the expression β instead of the term inside the second square root,

$$\beta = \sqrt{\frac{(\omega^2 LC - R_G) + \sqrt{(R_G + \omega^2 LC)^2}}{2}}$$

$$= \sqrt{\frac{\omega^2 LC - R_G + R_G + \omega^2 LC}{2}}$$

$$= \sqrt{\frac{2\omega^2 LC}{2}}$$

$$\boxed{\beta = \omega \sqrt{LC}}$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega \sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$\therefore v \rightarrow$ independent of frequency.

Subs $(R_G + \omega^2 LC)^2 \rightarrow$ in α expression,

$$\alpha = \sqrt{\frac{R_G - \omega^2 LC + \sqrt{(R_G + \omega^2 LC)^2}}{2}}$$

$$= \sqrt{\frac{R_G - \omega^2 LC + R_G + \omega^2 LC}{2}}$$

$$= \sqrt{\frac{2R_G}{2}} = \sqrt{R_G}$$

$$\boxed{\alpha = \sqrt{R_G}}$$

$\alpha \rightarrow$ independent of frequency.