Infinite Line

Input impedance – Two standard forms

The voltage & current expressions

$$E = E_R \cosh \sqrt{ZY}\, s + I_R Z_0 \sinh \sqrt{ZY}\, s$$

$$I = I_R \cosh \sqrt{ZY}\, s + \frac{E_R}{Z_0} \sinh \sqrt{ZY}\, s$$

subs $\gamma = \sqrt{ZY}$ & s by l, then $E = E_s$ & $I = I_s$

$$E_s = E_R \cosh \gamma l + I_R Z_0 \sinh \gamma l \rightarrow (3)$$

$$I_s = I_R \cosh \gamma l + \frac{E_R}{Z_0} \sinh \gamma l \rightarrow (4)$$

Input impedance

$$Z_s = \frac{E_s}{I_s}$$

$\therefore$ eq (3) $\div$ eq (4) $\rightarrow$

$$Z_s = \frac{E_s}{I_s} = \frac{E_R \cosh \gamma l + I_R Z_0 \sinh \gamma l}{I_R \cosh \gamma l + \frac{E_R}{Z_0} \sinh \gamma l}$$

$$= \frac{\cancel{I_R} Z_R \cosh \gamma l + \cancel{I_R} Z_0 \sinh \gamma l}{\frac{Z_0 \cancel{I_R} \cosh \gamma l + \cancel{I_R} Z_R \sinh \gamma l}{Z_0}}$$

$\div$ by I_R, to both Nr & Dr

$$\boxed{Z_s = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]} \rightarrow (5)$$

Other form of voltage & current expression,

$$E = \frac{E_R (Z_R + Z_0)}{2 Z_R}\left[e^{\sqrt{ZY}\,s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY}\,s}\right]$$

$$I = \frac{I_R (Z_R + Z_0)}{2 Z_0}\left[e^{\sqrt{ZY}\,s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY}\,s}\right]$$

Subs $\gamma = \sqrt{ZY}$; $s = l$, $E = E_S$ & $I = I_S$ in the above expressions,

$$E_S = \frac{E_R (Z_R + Z_0)}{2 Z_R}\left[e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}\right] \rightarrow (6)$$

$$I_S = \frac{I_R (Z_R + Z_0)}{2 Z_0}\left[e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}\right] \rightarrow (7)$$

eq (6) ÷ eq (7)

$$Z_S = \frac{E_S}{I_S} = \frac{\frac{\overset{I_R Z_R}{E_R}\cancel{(Z_R + Z_0)}}{\cancel{2} Z_R}\left[e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}\right]}{\frac{\cancel{I_R}\cancel{(Z_R + Z_0)}}{\cancel{2} Z_0}\left[e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}\right]}$$

$$\boxed{Z_S = Z_0 \left[\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}}\right]} \rightarrow (8)$$

Eqns. (5) & (8) are two standard forms of input impedance of a transmission line.

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Input impedances of open & short circuited lines

Input impedance

$$Z_s = Z_0 \left(\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right) \rightarrow (1)$$

For short circuiting the receiving side

$Z_R = 0$

$$\therefore Z_{sc} = Z_0 \left(\frac{\cancel{Z_0} \sinh \gamma l}{\cancel{Z_0} \cosh \gamma l} \right)$$

$$\boxed{Z_{sc} = Z_0 \tanh \gamma l} \rightarrow (2)$$

For open circuit, $\div$ by Z_R in eq (1)

$$Z_{oc} = Z_0 \left(\frac{\cosh \gamma l + Z_0/Z_R \sinh \gamma l}{\left(\frac{Z_0}{Z_R}\right) \cosh \gamma l + \sinh \gamma l} \right)$$

Subs $Z_R = \infty$ (open circuit)

$$Z_{oc} = Z_0 \left(\frac{\cosh \gamma l}{\sinh \gamma l} \right)$$

$$\boxed{Z_{oc} = Z_0 \coth \gamma l} \rightarrow (3)$$

$$\sqrt{Z_{sc} Z_{oc}} = Z_0 \tanh \gamma l \times Z_0 \coth \gamma l$$

$$= Z_0 \cancel{\tanh \gamma l} \times \frac{Z_0}{\cancel{\tanh \gamma l}} \quad \left(\because \coth = \frac{1}{\tanh}\right)$$

$$= Z_0^2$$

$$\therefore \boxed{Z_0 = \sqrt{Z_{sc} Z_{oc}}} \rightarrow (4)$$

$$\sqrt{\frac{Z_{sc}}{Z_{oc}}} = \sqrt{\frac{Z_0 \tanh \gamma l}{Z_0 \coth \gamma l}}$$

$$= \sqrt{\frac{\cancel{Z_0} \tanh \gamma l}{\frac{\cancel{Z_0}}{\tanh \gamma l}}}$$

$$= \sqrt{\tanh^2 \gamma l}$$

$$\therefore \boxed{\sqrt{\frac{Z_{sc}}{Z_{oc}}} = \tanh \gamma l}$$

$$\gamma l = \tanh^{-1}\left(\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right) \rightarrow (5)$$

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LOADING OF TRANSMISSION LINES

The use of inductance to increase L/C ratio to achieve distortionless conditions in transmission lines is called loading of transmission lines & the transmission lines are known as loaded lines.

Types of Loading

Loading is mainly done on telephone cables carrying voice signals.

Types of loading are

(i) continous loading
(ii) Lumped loading
(iii) Patch loading

(i) continous loading

* Inductance of the line is increased uniformly along the length of the line.
* A type of iron or some other high permeability magnetic material in the form of a wire or tape is wound around the copper conductors.
* This will increase the permeability of the surrounding medium, which in turn increases the inductance of the line.

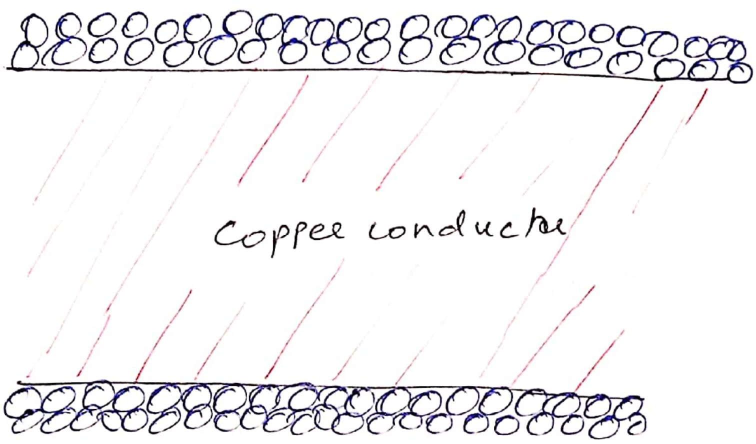
Advantages

(i) Attenuation is constant over a wide frequency range

(ii) Used only in submarine cables.

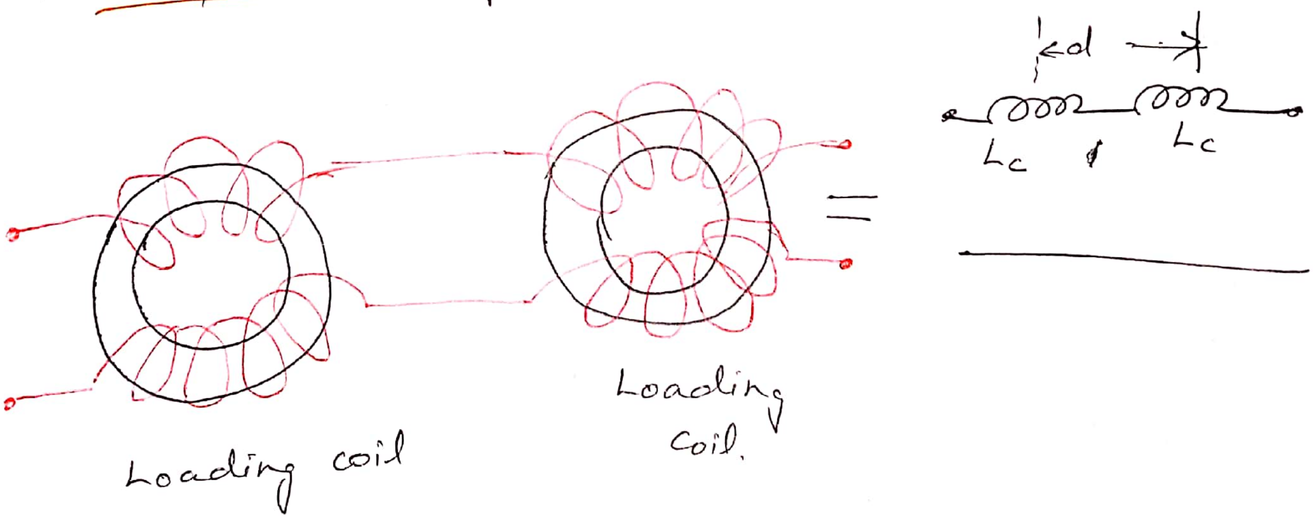
<u>Disadvantages</u>

(i) very expensive
(ii) Possibility of transmission delays
(iii) Eddy current and hysteresis losses increase with frequency, thus increasing the value of R



(Fig) Telephone cable with continous loading

(ii) <u>Lumped Loading</u>



(Fig) Lumped (or) coil loading

* The inductance coils are wound on a toroidal core and inserted periodically in series with the line.

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* This type of core produces coil of small dimention, high inductance and low eddy current losses.

Advantages:

(i) Large value of inductance is possible with reduced attenuation

(ii) Method of loading is more convenient.

(iii) Eddy current and hysteresis losses are less.

(iii) Patch loading

* This type of loading employs sections of continous loaded cable repeated by sections of unloaded cable.

* In submarine cables, patch loading is adequate to obtain the required reduction in attenuation

Advantages

(i) Advantage of loading is obtained

(ii) cost is greatly reduced

(iii) Reduction in attenuation.

Effect of loading on secondary constants

→ characteristic impedance increases

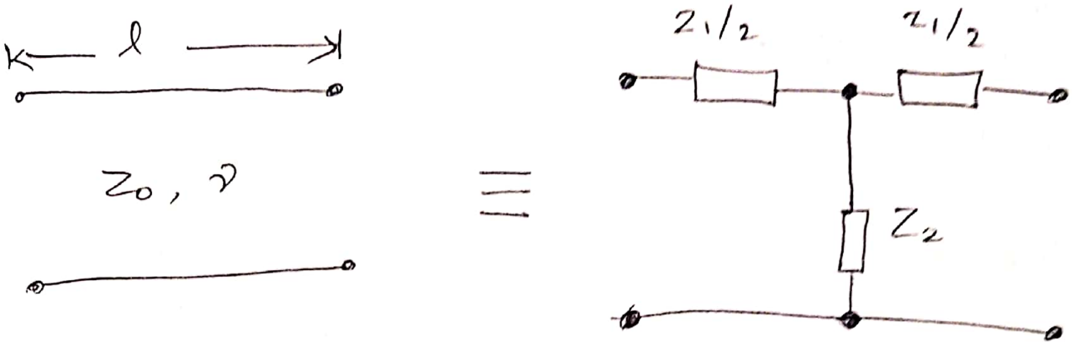
→ Attenuation constant is reduced

→ phase constant β is increased

→ phase velocity is reduced.

Campbell's Formula

* Consider a transmission line with characteristic impedance Z_0, propagation constant γ and length l.
* This transmission line can be represented by a symmetrical T - network.



(Fig) Equivalent circuit of an unloaded cable

$$\frac{Z_1}{2} = \frac{(R+j\omega L)l}{2} \quad \& \quad Z_2 = \frac{1}{(G+j\omega C)l}$$

For a symmetrical network,

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2} \quad \& \quad \sinh\gamma = \frac{Z_0}{Z_2}$$

Therefore, for a symmetrical T-network representing a length l of the line,

$$\cosh\gamma l = 1 + \frac{Z_1}{2Z_2} \rightarrow (1)$$

$$\sinh\gamma l = \frac{Z_0}{Z_2} \rightarrow (2)$$

$$\therefore \quad Z_2 = \frac{Z_0}{\sinh\gamma l} \rightarrow (3)$$

subs eq (3) in eq (1)

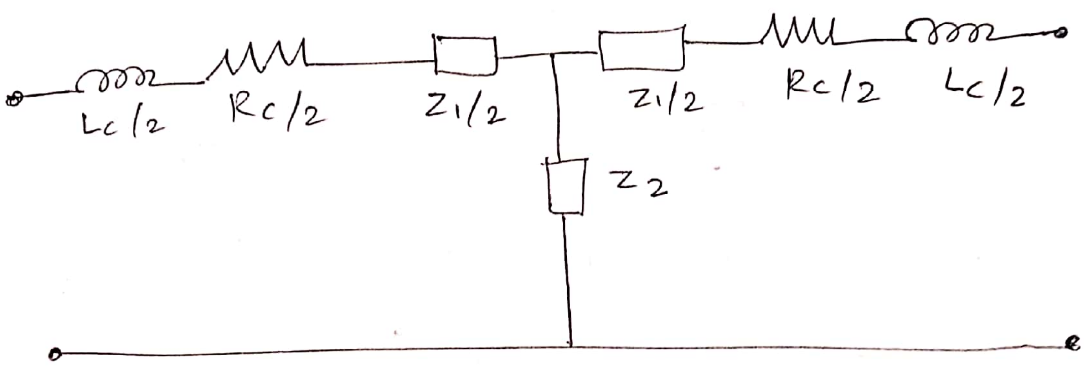
$$\cosh\gamma l = 1 + \frac{Z_1}{2} \times \frac{\sinh\gamma l}{Z_0}$$

$$\cosh \gamma l - 1 = \frac{Z_1 \sinh \gamma l}{2 Z_0}$$

$$\therefore Z_1 = \frac{2 Z_0 (\cosh \gamma l - 1)}{\sinh \gamma l} \rightarrow (4)$$

→ Now consider a loaded line with the inductance L_c & resistance R_c over the length L.

→ This loading can be split up into equal parts $\frac{L_c}{2}$ & $\frac{R_c}{2}$ in each arm of the T-network.



The equation for $\cosh \gamma l$ for the loaded line will be,

$$\cosh \gamma_L l = 1 + \frac{Z_1 + Z_c}{2 Z_2} \rightarrow (5)$$

γ_L → Propagation constant of a load line

$$Z_c = R_c + j\omega L_c.$$

Subs $Z_1 = 2 Z_0 \frac{(\cosh \gamma l - 1)}{\sinh \gamma l}$ & $Z_2 = \frac{Z_0}{\sinh \gamma l}$ $\rightarrow \cancel{(6)}$

in eq (5)

$$\cosh \gamma_L l = 1 + \frac{\frac{2 Z_0 (\cosh \gamma l - 1)}{\sinh \gamma l}}{\frac{2 Z_0}{\sinh \gamma l}} + \frac{Z_c}{\frac{2 Z_0}{\sinh \gamma l}}$$

$$= 1 + \cosh \gamma l - 1 + \frac{Z_c \sinh \gamma l}{2 Z_0}$$

$$\boxed{\cosh \gamma_L l = \cosh \gamma l + \frac{Z_c \sinh \gamma l}{2 Z_0}} \rightarrow (6)$$

* This equation (eq ⑥) is known as campbell's equation (or) campbell's formula.

* This equation is used to find the propagation constant of a loaded line &

* To study the effect of loading coils in reducing attenuation & distortion on the line.

Transfer Impedance

The input impedance of a transmission line,

$$Z_S = Z_0 \left(\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right) \rightarrow ①$$

In terms of exponentials

$$Z_S = Z_0 \left(\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}} \right) \rightarrow ②$$

Subs $k = \frac{Z_R - Z_0}{Z_R + Z_0}$

(Reflection co-efficient)

Eqn ② becomes

$$Z_S = Z_0 \left(\frac{e^{\gamma l} + k e^{-\gamma l}}{e^{\gamma l} - k e^{-\gamma l}} \right) \rightarrow ③$$

The voltage at the sending end

$$E_S = \frac{E_R (Z_R + Z_0)}{2 Z_R} \left(e^{\gamma l} + k e^{-\gamma l} \right) \rightarrow ④$$

$$= \frac{I_R \cancel{Z_R}\,(Z_R + Z_0)}{2\cancel{Z_R}} \left(e^{\gamma l} + K e^{-\gamma l}\right)$$

$$\left(\because E_R = I_R Z_R\right)$$

$$E_S = \frac{I_R (Z_R + Z_0)}{2} \left(e^{\gamma l} + K e^{-\gamma l}\right) \rightarrow (5)$$

The transfer impedance is

$$Z_T = \frac{E_S}{I_R}$$

From eqn. (5)

$$\frac{E_S}{I_R} = \frac{(Z_R + Z_0)}{2} \left(e^{\gamma l} + K e^{-\gamma l}\right)$$

Subs $K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$

$$\frac{E_S}{I_R} = Z_T = \frac{Z_R + Z_0}{2}\, e^{\gamma l} + \left(\frac{Z_R - Z_0}{\cancel{Z_R + Z_0}}\right) \times \frac{(\cancel{Z_R + Z_0})}{2}\, e^{-\gamma l}$$

$$Z_T = \frac{(Z_R + Z_0)}{2}\, e^{\gamma l} + \frac{(Z_R - Z_0)\, e^{-\gamma l}}{2}$$

$$= \frac{Z_R}{2} e^{\gamma l} + \frac{Z_0}{2} e^{\gamma l} + \frac{Z_R}{2} e^{-\gamma l} - \frac{Z_0}{2} e^{-\gamma l}$$

$$Z_T = Z_R \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)$$

$$\boxed{Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l} \rightarrow (6)$$

Eq (6) is the expression for transfer impedance of the transmission line.

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