TE WAVES ($E_z = 0$)

If $E_z = 0$, but $H_z \neq 0$ [$\therefore H_y$ & $E_x = 0$]

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\bar{\gamma}}{h^2} \frac{\partial H_z}{\partial x}$$

The wave equation

$$\frac{\partial^2 E_y}{\partial x^2} + \underbrace{\cancel{\frac{\partial^2 E_y}{\partial y^2}}}_{0} + \frac{\partial^2 E_y}{\partial z^2} = -\omega^2 \mu \varepsilon E_y \rightarrow (1)$$

$$\& \quad \frac{\partial^2}{\partial z^2} = \bar{\gamma}^2$$

Eq ① becomes,

$$\frac{\partial^2 E_y}{\partial x^2} + \bar{\gamma}^2 E_y = -\omega^2 \mu \varepsilon E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0 \qquad [\because h^2 = \bar{\gamma}^2 + \omega^2 \mu \varepsilon]$$

Since $E_y = E_y^{\circ} e^{-\bar{\gamma} z}$ the above eqn, becomes

$$\frac{\partial^2 E_y^{\circ}}{\partial x^2} + h^2 E_y^{\circ} = 0 \rightarrow ②$$

Eq ② is a differential Eqn. & the solution is

$$E_y^{\circ} = C_1 \sin hx + C_2 \cos hx \rightarrow ③$$

where C_1, C_2 are arbitrary constants.

Showing the variation in z direction

$$E_y = E_y^{\circ} e^{-\bar{\gamma} z} = (C_1 \sin hx + C_2 \cos hx) e^{-\bar{\gamma} z}.$$

C_1 & C_2 determined from boundary conditions.

Boundary condition

$E_{tan} = 0$ at the surface of the perfect conductors for all values of z and time.

i.e

$E_y = 0$ at $x = 0$

$E_y = 0$ at $x = a$

for all values of z.

Applying B.C (i)

$E_y = 0$ at $x = 0$

$$E_y^{\circ} = C_1 \sin 0 + C_2 \cos 0$$

$$E_y = C_2$$

$\therefore$ C_2 must be zero to make $E_y = 0$ at $x = 0$

Then Eqn. (3) becomes,

$$E_y^{\circ} = C_1 \sin hx \rightarrow (4)$$

Applying B.C (ii)

subs $E_y = 0$ at $x = a$ in eq (4)

$$E_y^{\circ} = C_1 \sin hx$$

To make $E_y = 0$, h must be equal to $\frac{m\pi}{a}$

$$\therefore h = \frac{m\pi}{a} \text{ for } m = 1, 2, 3 \ldots$$

$$\therefore E_y^{\circ} = C_1 \sin h\left(\frac{m\pi}{a}\right)$$

$$E_y = C_1 \sin h\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z} \rightarrow (5)$$

<u>Other Fields Determination</u>

$$\bar{\gamma} E_y = -j\omega\mu H_x$$

$$H_x = \frac{-\bar{\gamma}}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z} \rightarrow (6)$$

$$H_z = -\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial x}$$

$$H_z = -\frac{m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z} \rightarrow (7)$$

TEmo wave (or) mode

In eqns (5), (6) & (7)

→ Each value of m specifies a particular field configuration (or) mode.

→ The associated wave is known as TEmo wave (or) TEmo mode.