



For wave propagation,  $\vec{a} = \hat{z}$  ( $\vec{a} \cdot \vec{z} = 0$ )

$$E_y = c_1 \sin\left(\frac{m\pi}{a} x\right) e^{-j\beta z}$$

$$H_x = \frac{-j\beta}{j\omega\mu} c_1 \sin\left(\frac{m\pi}{a} x\right) e^{-j\beta z}$$

$$H_z = -\frac{m\pi}{j\omega\mu a} c_1 \cos\left(\frac{m\pi}{a} x\right) e^{-j\beta z}$$

TM waves ( $H_z = 0$ )

$H_x$  &  $E_y = 0$ ,  $E_z$ ,  $E_x$  &  $H_y$  will have value

$$H_y = (c_3 \sin hx + c_4 \cos hx) e^{-\gamma z} \quad (\text{solving the wave eqn for } H_y)$$

The boundary conditions can not be applied directly to  $H_y$  to evaluate the constants  $c_3$  &  $c_4$ , because tangential component of  $H_y$  is not zero at the surface of the perfect conductor.

$\therefore E_z$  is obtained in terms of  $H_y$  and then the boundary conditions applied to  $E_z$ .

$$\begin{aligned} \text{N.K.T } E_z &= \frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial x} \\ &= \frac{1}{j\omega\epsilon} \frac{\partial}{\partial x} [c_3 \sin hx + c_4 \cos hx] e^{-\gamma z} \\ &= \frac{1}{j\omega\epsilon} [c_3 \cosh x \cdot h - c_4 \sin hx \cdot h] e^{-\gamma z} \\ E_z &= \frac{h}{j\omega\epsilon} [c_3 \cosh x - c_4 \sin hx] e^{-\gamma z} \quad \text{--- (1)} \end{aligned}$$

Applying the boundary condition <sup>at</sup>  $x = 0$ ,  $E_z = 0$



subs  $x = 0$  in eq (1)

$$E_z = \frac{h}{j\omega\epsilon} [C_3 - 0]$$

$\therefore$  To satisfy the Boundary Condition I  $C_3$  must be zero.

$$\therefore C_3 = 0$$

eq (1) becomes

$$E_z = \frac{h}{j\omega\epsilon} [-C_4 \sin hx] e^{-\gamma z}$$

$$E_z = -\frac{h C_4}{j\omega\epsilon} \sin hx e^{-\gamma z} \rightarrow (2)$$

Applying B.C II

$E_z = 0$  at  $x = a$

subs  $x = a$  in eq (2)

$$E_z = -\frac{h C_4}{j\omega\epsilon} \sin ha e^{-\gamma z} \rightarrow (3)$$

To make  $E_z = 0$  at  $x = a$ ,  $h$  must be equal to  $\frac{m\pi}{a}$ , where  $m = 1, 2, 3, \dots$

$$\therefore h = \frac{m\pi}{a}, \text{ where } m = 1, 2, 3, \dots$$

subs  $h$  in eq (2)

$$E_z = -\frac{m\pi C_4}{a j\omega\epsilon} \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma z} \rightarrow (3)$$

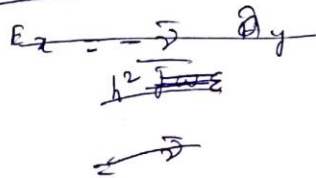
$$H_y = -\frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x}$$



$$H_y = \frac{-j\omega\epsilon}{h^2} \frac{\partial}{\partial x} \left[ -\frac{m\pi}{a} \frac{c_4}{j\omega\epsilon} \sin\left(\frac{m\pi}{a}\right) x e^{-\vec{\gamma}z} \right]$$

$$= -\frac{j\omega\epsilon}{h^2} x + \frac{m\pi}{a} \frac{c_4}{j\omega\epsilon} e^{-\vec{\gamma}z} \cos\left(\frac{m\pi}{a}\right) x \times \frac{m\pi}{a}$$

$$H_y = c_4 \cos\left(\frac{m\pi}{a}\right) x e^{-\vec{\gamma}z} \rightarrow (4)$$



$$E_x = -\vec{\gamma} \frac{\partial E_z}{\partial x}$$

$$E_x = -\frac{\vec{\gamma}}{h^2} \frac{\partial}{\partial x} \left[ -\frac{m\pi}{a} \frac{c_4}{j\omega\epsilon} \sin\left(\frac{m\pi}{a}\right) x e^{-\vec{\gamma}z} \right]$$

$$= -\frac{\vec{\gamma} x c_4}{m\pi j\omega\epsilon} \cos\left(\frac{m\pi}{a}\right) x \times \left(\frac{m\pi}{a}\right) e^{-\vec{\gamma}z}$$

$$E_x = +\frac{\vec{\gamma}}{j\omega\epsilon} c_4 \cos\left(\frac{m\pi}{a}\right) x e^{-\vec{\gamma}z}$$

Subs  $\vec{\gamma} = j\beta$  for wave propagation, the field equations become,

$$E_z = -\frac{m\pi}{a} \frac{c_4}{j\omega\epsilon} \sin\left(\frac{m\pi}{a}\right) x e^{-j\beta z} \rightarrow (6)$$

$$H_y = c_4 \cos\left(\frac{m\pi}{a}\right) x e^{-j\beta z} \rightarrow (7)$$

$$E_x = \frac{j\beta}{j\omega\epsilon} c_4 \cos\left(\frac{m\pi}{a}\right) x e^{-j\beta z}$$

$$E_x = \frac{\beta}{\omega\epsilon} c_4 \cos\left(\frac{m\pi}{a}\right) x e^{-j\beta z} \rightarrow (8)$$

Eqs (6), (7), (8) are field equations of TM waves between parallel planes.



Infinite number of modes are possible for the various values of  $m$  from 1 to  $\infty$ .

$m=0$  does not make all the fields vanish.