For wave propagation, $\bar{\gamma} = j\bar{\beta}$ $(\bar{\alpha} = 0)$

$$E_y = c_1 \sin\left(\frac{m\pi}{a} x\right) e^{-j\bar{\beta} z}$$

$$H_x = \frac{-j\bar{\beta}}{j\omega\mu} c_1 \sin\left(\frac{m\pi}{a} x\right) e^{-j\bar{\beta} z}$$

$$H_z = -\frac{m\pi}{j\omega\mu a} c_1 \cos\left(\frac{m\pi}{a} x\right) e^{-j\bar{\beta} z}$$

TM waves $(H_z = 0)$

H_x & $E_y = 0$, E_z, E_x & H_y will have value

$$H_y = (c_3 \sin hx + c_4 \cos hx) e^{-\bar{\gamma} z}$$ (solving the wave eqn for H_y)

The boundary conditions can not be applied directly to H_y to evaluate the constants c_3 & c_4, because tangential component of H_y is not zero at the surface of the perfect conductor.

$\therefore$ E_z is obtained in terms of H_y and then the boundary conditions applied to E_z.

W.K.T $\quad E_z = \frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial x}$

$$= \frac{1}{j\omega\varepsilon} \frac{\partial}{\partial x} [c_3 \sin hx + c_4 \cos hx] e^{-\bar{\gamma} z}$$

$$= \frac{1}{j\omega\varepsilon} [c_3 \cos hx \cdot h - c_4 \sin hx \cdot h] e^{-\bar{\gamma} z}$$

$$E_z = \frac{h}{j\omega\varepsilon} [c_3 \cos hx - c_4 \sin hx] e^{-\bar{\gamma} z} \quad \text{—(1)}$$

Applying the Boundary condition at $x = 0$, $E_z = 0$

subs $x = 0$ in eq (1)

$$E_z = \frac{h}{j\omega\varepsilon}\left[C_3 - 0\right]$$

$\therefore$ To satisfy the Boundary condition I C_3 must be zero.

$$\therefore \boxed{C_3 = 0}$$

eq (1) becomes

$$E_z = \frac{h}{j\omega\varepsilon}\left[-C_4 \sin hx\right]e^{-\bar{\jmath}z}$$

$$E_z = -\frac{hC_4}{j\omega\varepsilon}\sin hx\, e^{-\bar{\jmath}z} \rightarrow (2)$$

Applying B.C II

$E_z = 0$ at $x = a$

subs $x = a$ in eq (2)

$$E_z = -\frac{a\, C_4}{j\omega\varepsilon}\sin ha\, e^{-\bar{\jmath}z} \cancel{\rightarrow (3)}$$

To make $E_z = 0$ at $x = a$, h must be equal to $\frac{m\pi}{a}$, where $m = 1, 2, 3 \ldots$

$$\therefore h = \frac{m\pi}{a}, \text{ where } m = 1, 2, 3 \ldots$$

subs in eq (2)

$$\boxed{E_z = -\frac{m\pi}{a}\frac{C_4}{j\omega\varepsilon}\sin\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\jmath}z}} \rightarrow (3)$$

$$H_y = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial}{\partial x}\left[-\frac{m\pi}{a}\frac{C_4}{j\omega\varepsilon}\sin\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z}\right]$$

$$= -\frac{\cancel{j\omega\varepsilon}}{\cancel{h^2}} \times +\frac{m\cancel{\pi}}{a}\frac{C_4}{\cancel{j\omega\varepsilon}} e^{-\bar{\gamma} z}\cos\left(\frac{m\pi}{a}\right)x \times \frac{m\cancel{\pi}}{\cancel{a}}$$

$$\boxed{H_y = C_4\cos\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z}} \rightarrow (4)$$

$$\cancel{E_z = \frac{-\bar{\gamma}}{h^2 j\omega\varepsilon}\partial_y}$$

$$E_x = \frac{-\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\bar{\gamma}}{h^2}\frac{\partial}{\partial x}\left[-\frac{m\pi}{a}\frac{C_4}{j\omega\varepsilon}\sin\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z}\right]$$

$$= -\frac{\bar{\gamma}\alpha\, C_4}{m\pi\, j\omega\varepsilon}\cos\left(\frac{m\pi}{a}\right)x \times \left(\frac{m\pi}{a}\right)e^{-\bar{\gamma} z}$$

$$\boxed{E_x = +\frac{\bar{\gamma}\, C_4}{j\omega\varepsilon}\cos\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z}}$$

Subs $\bar{\gamma} = j\beta$ for wave propagation, the field equations become,

$$\boxed{E_z = -\frac{m\pi}{a}\frac{C_4}{j\omega\varepsilon}\sin\left(\frac{m\pi}{a}\right)x\, e^{-j\bar{\beta} z}} \rightarrow (6)$$

$$\boxed{H_y = C_4\cos\left(\frac{m\pi}{a}\right)x\, e^{-j\bar{\beta} z}} \rightarrow (7)$$

$$E_x = \frac{j\bar{\beta}}{j\omega\varepsilon}C_4\cos\left(\frac{m\pi}{a}\right)x\, e^{-j\bar{\beta} z}$$

$$\boxed{E_x = \frac{\bar{\beta}}{\omega\varepsilon}C_4\cos\left(\frac{m\pi}{a}\right)x\, e^{-j\bar{\beta} z}} \rightarrow (8)$$

Eqns (6), (7), (8) are field equations of TM waves between parallel planes.

Infinite number of modes are possible for the various values of m from 1 to ∞.

m = 0 does not make all the fields vanish.