The Smith chart

Smith chart is a modified form of circle diagrams by P.H Smith.

It is obtained from the eqn!

$$\left(\frac{S-1}{S+1}\right)\angle\phi - 2\beta s = |K| \angle\phi - 2\beta s = \frac{r_a^2 - 1 + x_a^2 + j2x_a}{(r_a+1)^2 + x_a^2}$$

Introducing new variables, $U + jV$

$$U + jV = \frac{r_a^2 - 1 + x_a^2}{(r_a+1)^2 + x_a^2} + j\frac{2x_a}{(r_a+1)^2 + x_a^2}$$

Elimination of first x_a then r_a gives two eqns,

$$\left[U - \left(\frac{r_a}{r_a+1}\right)^2\right] + V^2 = \frac{1}{(r_a+1)^2} \rightarrow (1)$$

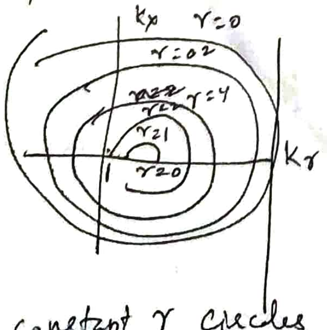
$$(U-1)^2 + \left(V - \frac{1}{x_a}\right)^2 = \frac{1}{x_a^2} \rightarrow (2)$$

Eqn. (1) represents a family of constant r_a circles having centres on the U axis at r_a/r_a+1 & radii of $1/(r_a+1)$.

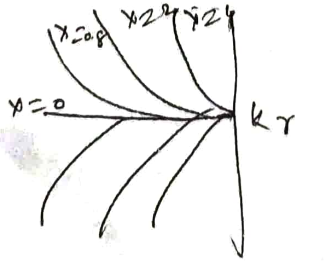
Eqn (2) represents a family of constant x_a circles with centres at $1 + j/x_a$ & radii equal to $1/x_a$.

The two families of circles gives smith chart.

Maxl. value of $U + jV$ is fixed at unity by the maxl. value of k.

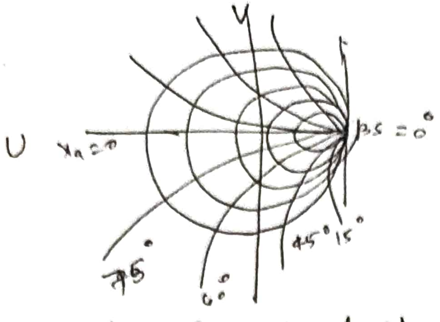


Constant r circles



Constant x circles

11



Smith circle diagram

Properties of smith chart

1) The smith chart is used for impedances as well as admittances.

2) The smith chart consists of constant r_i circles and constant x_i circles superpositioned at one chart. The values of r_i and x_i are normalized, given by

$$r_i = \frac{R}{R_0} \quad \& \quad x_i = \frac{j x_R}{R_0}$$

3) The constant r_i circles have their centres on the horizontal axis & constant x_i circles have their centres on vertical axis.

4) The smith chart is based on the assumption the

$$|k| \angle \phi - 2\beta s = U + jV$$

5) Centre of the smith chart is (1,0)

6) Horizontal line on the smith chart represents real axis (r_i axis) for impedance plot (or) g_i for admittance chart.

7) At extreme left $r_i = 0$ & $x_i = 0$ is short circuit condition &

At extreme right $r_i = \infty$ indicates infinite impedance (or) open circuit condition.

8) The outer rim of the chart is scaled in wavelengths.

9) The complete length of the smith chart is $\lambda/2$.

10) wavelength toward generator - clockwise direction
wavelength toward load - anticlockwise direction

11) If the smith chart is used for impedances the inductive reactance is above real axis & capacitive reactance is below real axis.

V_{max} - right
V_{min} - left.

Applications of smith chart

Problem: consider a 30 m long lossless transmission line with the characteristic impedance of 50 Ω operating at 2 MHZ. If the line is terminated in impedance 60+j40 Ω. calculate

K, S, Zin, ZL, if the velocity on the line is V = 0.6c

1) Plotting an impedance

$$Z_R = 60 + j40\ \Omega$$

Normalized impedance $\frac{Z_R}{R_0} = \frac{60+j40}{50} = 1.2 + j0.8\ \Omega.$

Locate point P on Smith chart, where real part is 1.2 & imaginary part 0.8 meet together

2) VSWR

After plotting normalized impedance the value of SWR is obtained by drawing S circle with the centre of the chart and radius equal to distance between O & P.

The circle cuts the real axis at right side gives the value of SWR. $S = 2.1$

3) <u>Reflection coefficient K</u>

To find the value of K, extend the line from centre of the chart through point P to the outer circle of the chart.

The point at which the line cuts the outer rim gives the angle of K.

To find the magnitude of K, measure the distance between O to P.

The K scale is provided at the bottom of the Smith chart from the centre, draw an arc with distance equal to OP gives the magnitude of K.

$|K| = 0.35$, $\phi = 55.5$

$K = 0.35 \angle 55.5^{\circ}$

4) <u>Input impedance</u>

From load impedance, move towards generator (in clockwise direction) for the distance equal to the length of the line, we get input impedance point.

Here in the problem $l = 30$ m.

l in wavelengths $\left(\frac{30}{90}\right) = 0.333\lambda$.

$$\lambda = \frac{v}{f} = \frac{0.6 \times 3 \times 10^{8}}{2 \times 10^{6}} = 90 \text{ m.}$$

$$\frac{Z_{in}}{R_0} = 0.48 + j0.035$$

$$Z_{in} = (0.48 + j0.035) \times 50 = 24 + j1.75 \text{ ohms.}$$

Impedance to Admittance conversion

After getting normalized impedance the diametrically opposite point on the circle gives the value of admittance.

Load admittance $\frac{Y_R}{G_0} = 0.58 - j0.4$

$$Y_R = \frac{1}{R_0} \times y_R$$

$$= \frac{0.58 - j0.4}{50}$$

$$y_R = 0.016 - j8 \times 10^{-3} \text{ mhos.}$$

Applications

1) It is used to find input impedance of a transmission line.

2) used to find reflection ~~coef~~ coefficient.

3) It is useful for finding impedance from admittance and viceversa.

4) It is used ~~fo~~ to find SWR.

5) It is used to design stub for impedance matching.