SINGLE STUB MATCHING

Need for stub matching

When a high frequency line is operated/terminated in its characteristic impedance R_0, it is operated as a smooth line.

Under such conditions there will be no reflection and hence we get maximum power delivered to the load and increased efficiency.

But in practice the loads such as antennas do not provide resistances equal to R_0 of the line. Hence it is necessary to add some impedance matching sections between the line and the load, such that load appears as a resistance R_0 of the line.

One of the method of impedance matching is to use open (or) short stub lines. In this method a stub of suitable length is connected in parallel with the line at a certain distance from the load.

Because of paralleling the stub it is convenient to work with admittances.

Principle

The input ~~impedance~~ admittance Y_s looking towards the load at any point on the line is given by

$$Y_s = G_0 \pm jB$$

Then the short circuited stub of input susceptance $\mp jB$ is connected at that point across the transmission line.

Then the total admittance is given by,

$$Y_s = G_0 \pm \cancel{jB} \mp \cancel{jB}$$

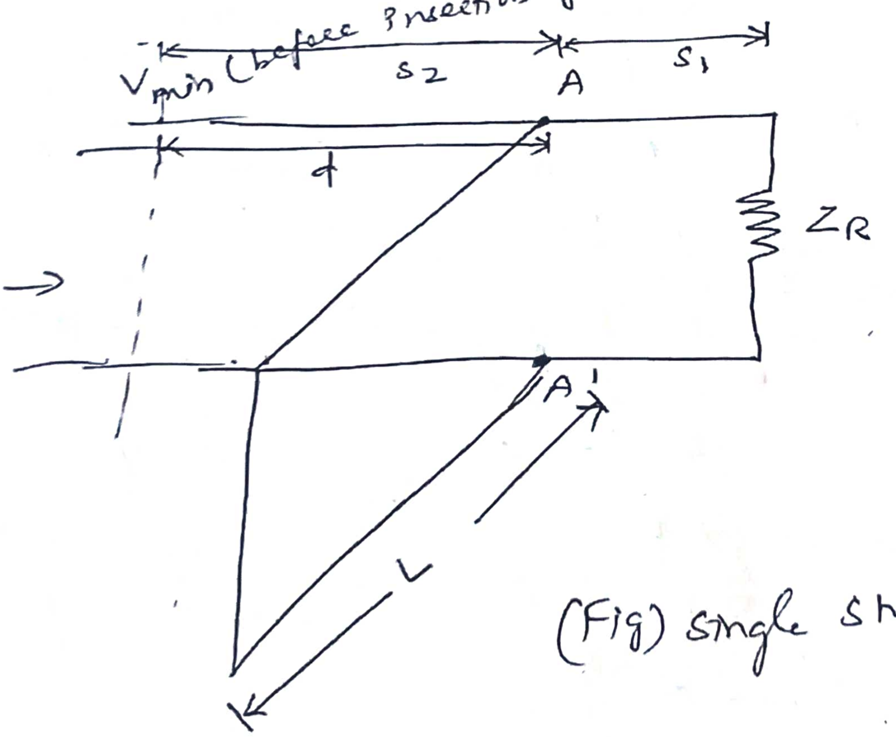
$$Y_s = G_0$$

$$Z_s = R_0$$

The input impedance of the line at a point looking towards load is

$$Z_s = \frac{1}{Y_s} = R_0$$

Thus the line from the source to the point is then terminated in R_0. It acts as a smooth line.



(Fig) single stub matching.

2 Design Parameters

(i) The point of stub connection

(ii) length of the stub.

...pedance of the line at the stub location

The input impedance expression is given by,

$$Z_{in} = Z_s = R_0 \left[\frac{1 + |K| \angle \phi - 2\beta s}{1 - |K| \angle \phi - 2\beta s}\right] \rightarrow (1)$$

The input admittance

$$Y_s = \frac{1}{R_0}\left[\frac{1 - |K| \angle \phi - 2\beta s}{1 + |K| \angle \phi - 2\beta s}\right] \rightarrow (3)$$

In rectangular co-ordinates.

$$Y_s = G_s + jB_s = \frac{1}{R_0}\left[\frac{1 - [|K| \cos(\phi - 2\beta s) + j|K| \sin(\phi - 2\beta s)]}{1 + [|K| \cos(\phi - 2\beta s) + j|K| \sin(\phi - 2\beta s)]}\right]$$

$$G_s + jB_s = G_0\left[\frac{1 - |K| \cos(\phi - 2\beta s) - j|K| \sin(\phi - 2\beta s)}{1 + |K| \cos(\phi - 2\beta s) + j|K| \sin(\phi - 2\beta s)}\right]$$

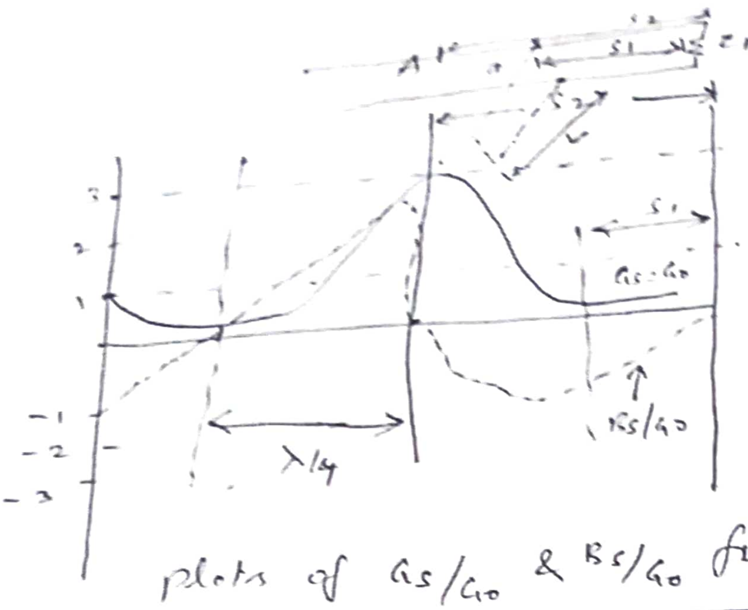
Rationalizing

$$G_s + jB_s = G_0\left[\frac{1 - |K| \cos(\phi - 2\beta s) - j|K| \sin(\phi - 2\beta s)}{1 + |K| \cos(\phi - 2\beta s) + j|K| \sin(\phi - 2\beta s)}\right] \times \frac{[1 + |K| \cos(\phi - 2\beta s) - j|K| \sin(\phi - 2\beta s)]}{[1 + |K| \cos(\phi - 2\beta s) - j|K| \sin(\phi - 2\beta s)]}$$

simplifying

$$G_s + jB_s = G_0\left[\frac{1 - |K|^2 - j2|K| \sin(\phi - 2\beta s)}{1 + |K|^2 + 2|K| \cos(\phi - 2\beta s)}\right]$$

$$\frac{G_s}{G_0} + j\frac{B_s}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K| \cos(\phi - 2\beta s)} - \frac{j2|K| \sin(\phi - 2\beta s)}{1 + |K|^2 + 2|K| \cos(\phi - 2\beta s)}$$



plots of G_s/G_0 & B_s/G_0 for $|K| = 0.5$

From the plot G_s/G_0 is max for the value of s_2 at which the cosine term is -1

$$(\phi - 2\beta s_2) = -\pi$$

$$s_2 = \frac{\phi + \pi}{2\beta} \rightarrow (3)$$

At s_2 the maxl. value of G_s/G_0 is given by,

$$\left(\frac{G_s}{G_0}\right)_{max} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K|(-1)} = \frac{1 - |K|^2}{1 + |K|^2 - 2|K|}$$

$$= \frac{(1 + |K|)\cancel{(1 - |K|)}}{(1 - |K|)^{\cancel{2}}}$$

$$= \frac{1 + |K|}{1 - |K|} = S$$

$$\frac{G_s}{G_0} = \frac{Y_{RS}}{Y_{R0}} = \frac{R_0}{R_S} = S$$

~~$R_S = R_S/R_0$~~ $\boxed{R_S = R_0/S}$

The input susceptance of the line at the stub location nearest to the load,

$$B_s = G_0 \left[\frac{-2|K| \sin\left(\phi - 2\beta \times \frac{\phi + \pi \mp \cos^{-1}(|K|)}{2\beta}\right)}{1 + |K|^2 + 2|K| \cos\left(\phi - 2\beta * \frac{\phi + \pi - \cos^{-1}(|K|)}{2\beta}\right)} \right]$$

$$= G_0 \left[\frac{-2|K| \sin(\pi + \cos^{-1}(|K|))}{1 + |K|^2 + 2|K| \cos(\pi + \cos^{-1}(|K|))} \right]$$

At an angle whose cosine is $|K|$, the sine is $\sqrt{1-|K|^2}$.

$$B_s = G_0 \left(\frac{2|K| \sqrt{1-|K|^2}}{1-|K|^2} \right) = G_0 \left(\frac{2|K|}{\sqrt{1-|K|^2}} \right)$$

The susceptance of a short circuited stub,

$$B_{sc} = -G_0 \cot \beta L$$

$L \rightarrow$ length of the short circuited stub.

If the line have value equal to G_0

$$\frac{G_0}{\tan \beta L} = G_0 \left(\frac{2|K| \sqrt{1-|K|^2}}{1-|K|^2} \right)$$

$$\boxed{L = \frac{\lambda}{2\pi} \tan^{-1} \left(\frac{\sqrt{1-|K|^2}}{2|K|} \right)}$$

(or) $$L = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{S}}{S-1}$$

This is the length of the stub to be placed d meters toward the load from a point at which a V_{min} existed before attachement of the stub

Thus at a point s_2 from load, input impedance is resistive & its value equal to R_0/S. This the point of min. voltage at distance s_2 from load.

At s_1 from load, $G_s = G_0$
This is the point at which the stub is to be connected.

$$\frac{G_s}{G_0} = 1 = \frac{1-|K|^2}{1+|K|^2+2|K|\cos(\phi-2\beta s_1)}$$

$$\cancel{1}-|K|^2 = \cancel{1}+|K|^2+2|K|\cos(\phi-2\beta s_1)$$

$$\cancel{2}|K|^{\cancel{2}} = \cancel{2}|\cancel{K}|\cos(\phi-2\beta s_1)$$

$$\phi-2\beta s_1 = \cos^{-1}(-|K|)$$

$$\phi-2\beta s_1 = -\pi + \cos^{-1}(|K|)$$

$$2\beta s_1 = \phi+\pi \mp \cos^{-1}(|K|)$$

$$s_1 = \frac{\phi+\pi \mp \cos^{-1}(|K|)}{2\beta}$$

$$d = s_2 - s_1$$

$$d = \frac{\phi+\pi}{2\beta} - \left(\frac{\phi+\pi \mp \cos^{-1}(|K|)}{2\beta}\right)$$

$$= \frac{\pm\cos^{-1}(|K|)}{2\beta} = \frac{\pm\cos^{-1}(|K|)}{2\times 2\pi/\lambda}$$

$$\boxed{d = \frac{\pm\cos^{-1}\left(\frac{S-1}{S+1}\right)\frac{\lambda}{4}}{\pi}}$$

The stub should be connected at this distance 'd' measured from either direction from a V_{min} nearest to the load.