SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade (3rd Cycle)
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT302 – TRANSMISSION LINES AND ANTENNAS

III YEAR/ V SEMESTER

UNIT 1 – TRANSMISSION LINETHEORY

TOPIC 2 – GENERAL SOLUTION OF TRANSMISSION LINE

WHAT IS THE DIFFERENCE BETWEEN THESE CIRCUITS ?

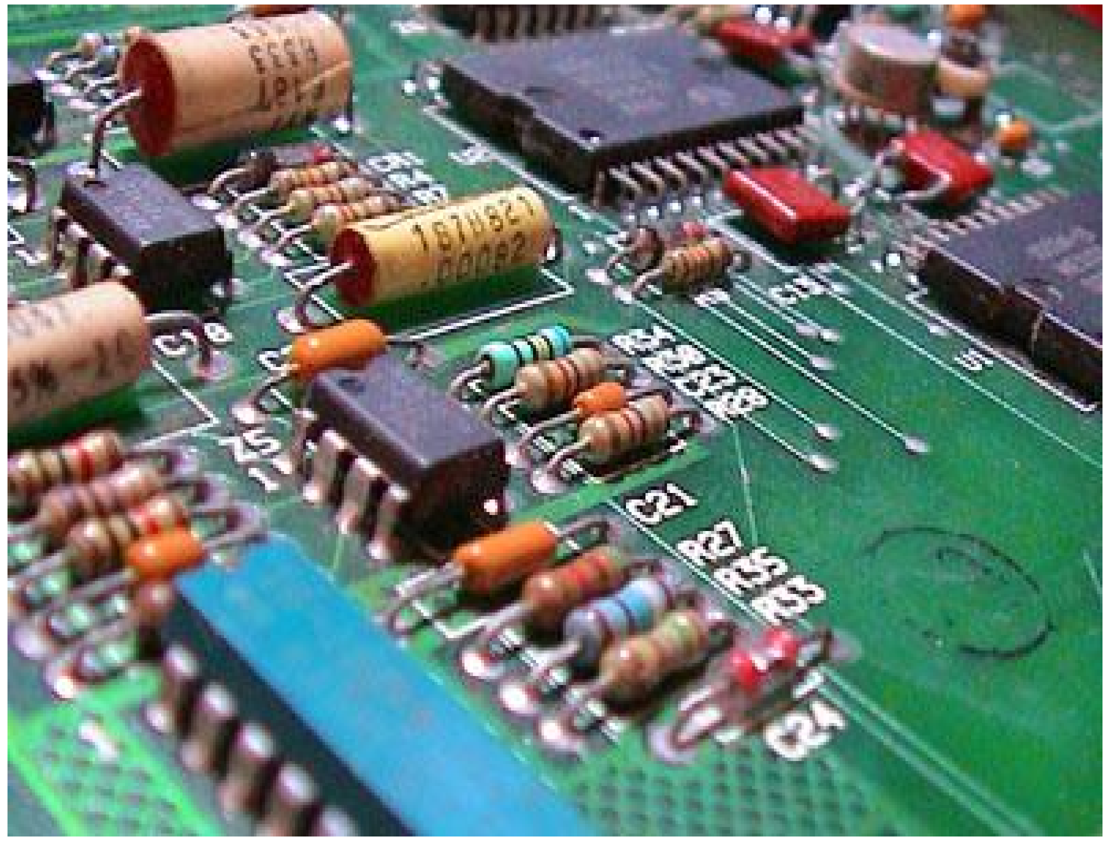


FIG 1. ELECTRONIC CIRCUIT

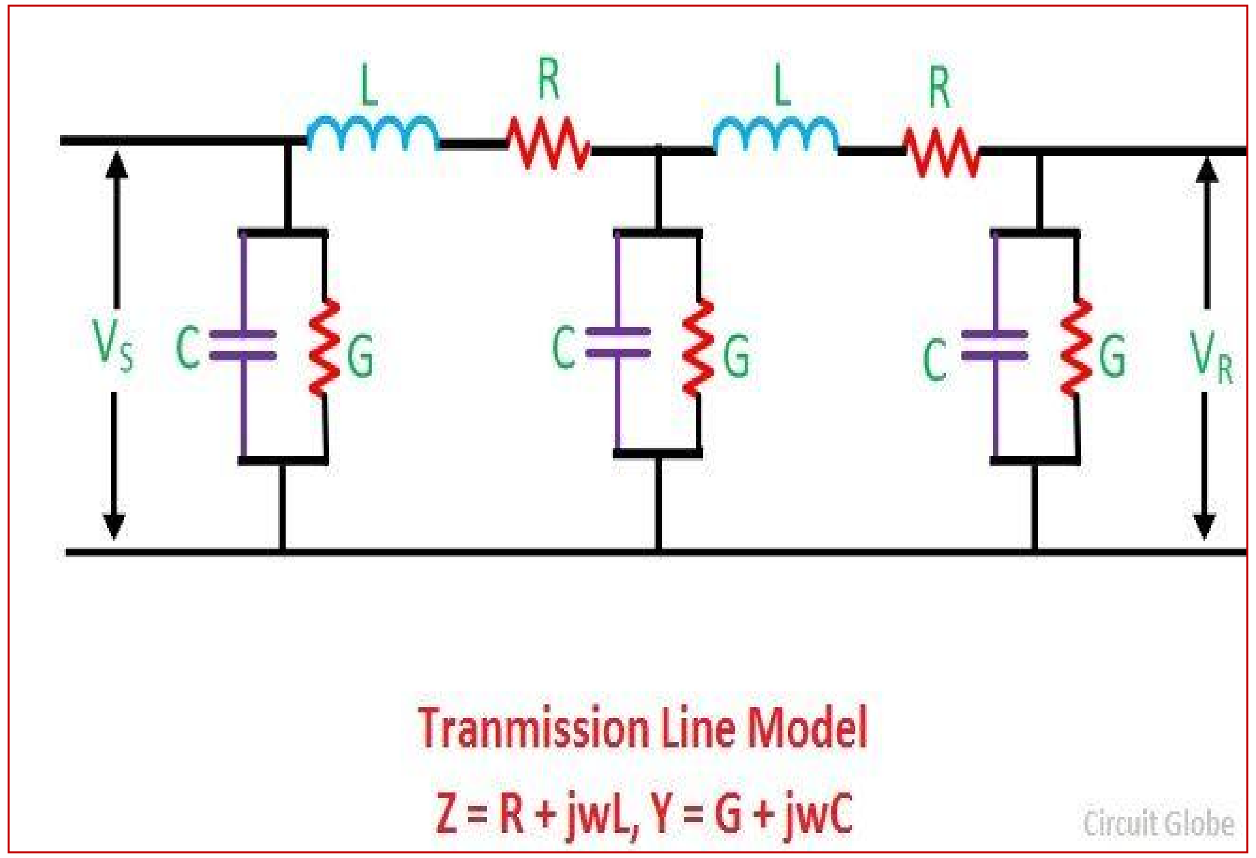


FIG 2. TRANSMISSION LINE MODEL

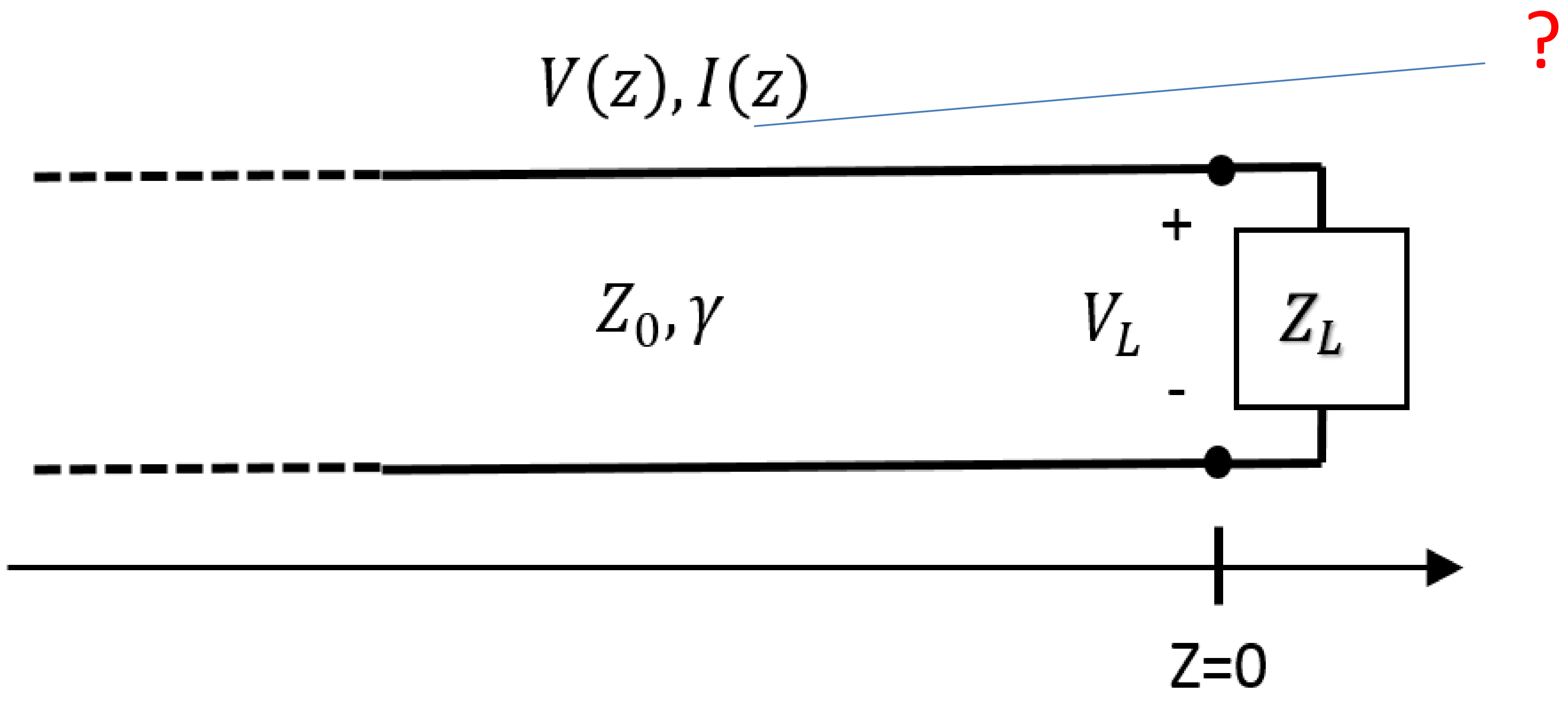
LUMPED & DISTRIBUTED NETWORKS

LUMPED NETWORK (FIG 1)	DISTRIBUTED NETWORK (FIG 2)
A network which is formed by lumped components like resistors, capacitors and inductors.	A network which is formed by using sections of transmission lies.
Parameters can be easily determines because they are fixed at discrete points in the circuit.	Parameters are distributed throughout the length of a transmission line.
One can easily recognize the presence of the components like resistor, capacitor and inductor.	It is difficult to recognize the presence of the components like resistor, capacitor and inductor.
EX. Ordinary electric circuits	Ex. Transmission Lines

TRANSMISSION LINE GENERAL SOLUTION

Transmission line General solution is used to

- Find voltage and current at any point on a line



TRANSMISSION LINE GENERAL SOLUTION

Line parameters

R= series resistance, ohms per unit length of line
(includes both wires)

L= series inductance, henrys per unit length of line

C= capacitance between conductors, faradays per unit length of line

G= shunt leakage conductance between conductors, mhos per unit length of line

Z =series impedance = $R+j\omega L$

ωL = series reactance, ohms per unit length of line

TRANSMISSION LINE GENERAL SOLUTION

Line parameters

Y = shunt admittance, ohms per unit length of line

$$Y = G + j\omega C$$

ωC = shunt susceptance, mhos per unit length of line

S = distance to the point of observation, measured from the receiving end of the line

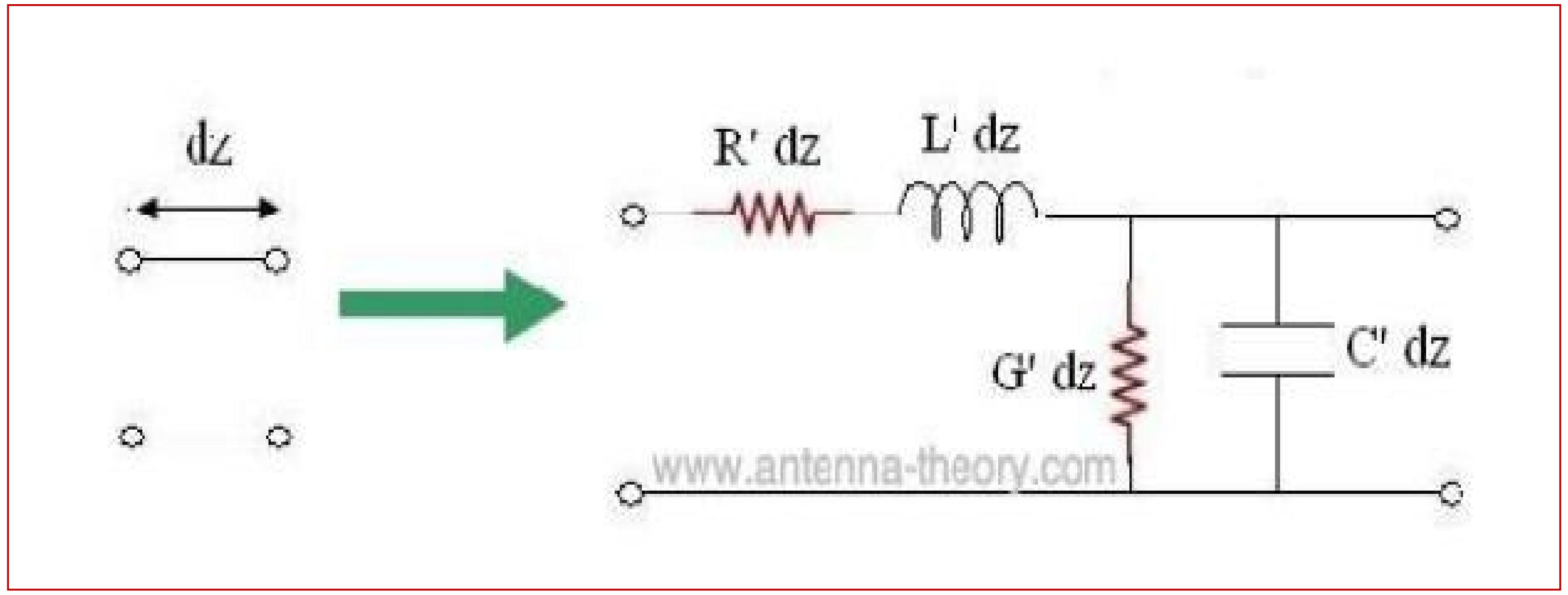
I = Current in the line at any point

E= voltage between conductors at any point

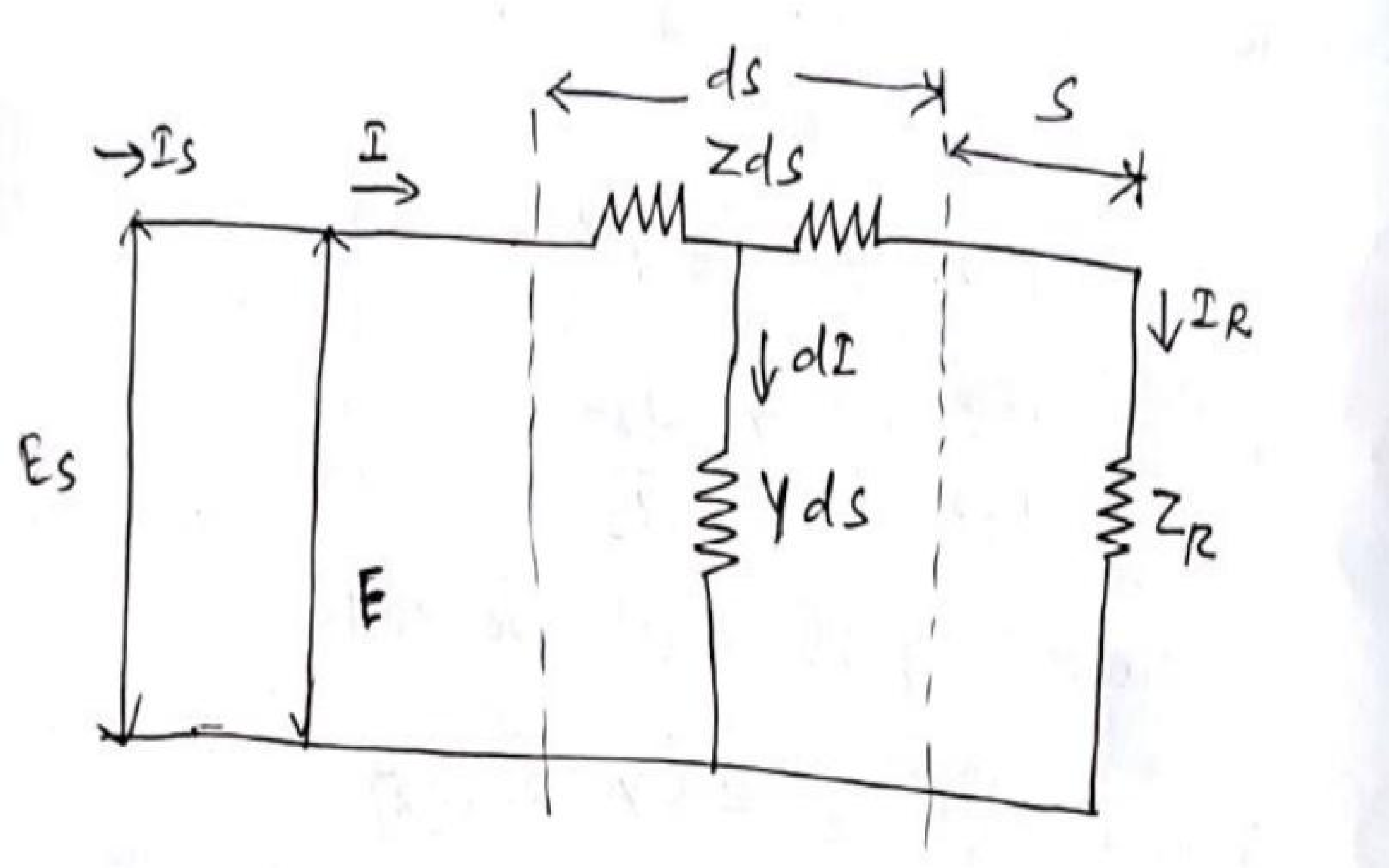
l = Length of the line

TRANSMISSION LINE GENERAL SOLUTION

- For finding Transmission line general solution a small section of a long transmission line is taken as shown in the diagram.
- Then modelled each small segment with a small series resistance, series inductance, shunt conductance, and shunt capacitance:



TRANSMISSION LINE GENERAL SOLUTION



TRANSMISSION LINE GENERAL SOLUTION

Voltage drop in the Line

- This incremental section is of length of ds and carries a current I.
- The series line impedance being Z ohms and the voltage drop in the length ds is

$$dE = IZ\,ds \qquad \text{-------------} \qquad (1)$$

$$\frac{dE}{ds} = IZ \qquad \text{-------------} \qquad (2)$$

TRANSMISSION LINE GENERAL SOLUTION

Current across the Line

- The admittance of the line is Yds mhos.
- The current dI that follows across the line or from one conductor to the other is

$$dI = EYds \qquad \text{-------------} \qquad (3)$$

$$\frac{dI}{ds} = EY \qquad \text{-------------} \qquad (4)$$

TRANSMISSION LINE GENERAL SOLUTION

The equations 2 and 4 are differentiated with respect to "s"

$$\frac{d^2E}{ds^2} = Z\frac{dI}{ds} \rightarrow (5)$$

subs eq (4) in eq (5)

$$\frac{d^2E}{ds^2} = ZEY \rightarrow (6)$$

$$\frac{d^2I}{ds^2} = Y\frac{dE}{ds} \rightarrow (7)$$

subs eq (2)

$$\frac{d^2I}{ds^2} = YIZ \rightarrow (8)$$

TRANSMISSION LINE GENERAL SOLUTION

$$(m^2 - ZY)E = 0$$

$$m^2 - ZY = 0$$

$$m^2 = ZY$$

$$m = \pm\sqrt{ZY}$$

$$E = Ae^{\sqrt{ZY}s} + Be^{-\sqrt{ZY}s} \rightarrow (9)$$

$$I = Ce^{\sqrt{ZY}s} + De^{-\sqrt{ZY}s} \rightarrow (10)$$

Where A,B,C,D are arbitrary constants of integration.

TRANSMISSION LINE GENERAL SOLUTION

Assigning conditions to find the solution

Since the distance is measured from the receiving end of the line

$$s=0,\ I=I_R\ \&\ E=E_R$$

Then equations (9) & (10) becomes

$$E_R = A+B \qquad \text{---------}\ (11)$$

$$I_R = C+D \qquad \text{---------}\ (12)$$

TRANSMISSION LINE GENERAL SOLUTION

$$\frac{dE}{ds} = A\sqrt{ZY}\, e^{\sqrt{ZY}\, s} - B e^{-\sqrt{ZY}\, s} \times \sqrt{ZY} \rightarrow (13)$$

$$\frac{dI}{ds} = C\sqrt{ZY}\, e^{\sqrt{ZY}\, s} - D\sqrt{ZY}\, e^{-\sqrt{ZY}\, s} \rightarrow (14)$$

$$IZ = A\sqrt{ZY}\, e^{\sqrt{ZY}\, s} - B e^{-\sqrt{ZY}\, s} \cdot \sqrt{ZY} \qquad \left(\frac{dE}{ds} = IZ\right)$$

$$I = A\sqrt{\frac{Y}{Z}}\, e^{\sqrt{ZY}\, s} - B\sqrt{\frac{Y}{Z}}\, e^{-\sqrt{ZY}\, s} \rightarrow (15)$$

$$\text{IIIly eq (14)} \Rightarrow E = C\sqrt{\frac{Z}{Y}}\, e^{\sqrt{ZY}\, s} - D\sqrt{\frac{Z}{Y}}\, e^{-\sqrt{ZY}\, s} \rightarrow (16)$$

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TRANSMISSION LINE GENERAL SOLUTION

Eqs (15) & (16) $\quad s = 0, \quad E = E_R \;\&\; I = I_R$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{Y/Z} \rightarrow (17)$$

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \rightarrow (18)$$

Multiply eq (12) by $\sqrt{\frac{Z}{Y}}$

$$I_R\sqrt{\frac{Z}{Y}} = C\sqrt{\frac{Z}{Y}} + \cancel{D\sqrt{\frac{Z}{Y}}}$$

eq (18) $\quad E_R = C\sqrt{\frac{Z}{Y}} - \cancel{D}\sqrt{\frac{Z}{Y}}$

(+) $\qquad$ Add

$$E_R + I_R\sqrt{\frac{Z}{Y}} = 2C\sqrt{\frac{Z}{Y}}$$

$$\boxed{C = \frac{E_R}{2}\sqrt{\frac{Y}{Z}} + I_R}$$

A, B, D also.

TRANSMISSION LINE GENERAL SOLUTION

$\frac{E_R}{Z_R}$

$$A = \frac{E_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)$$

$$\sqrt{Z/Y} = Z_0$$

$$B = \frac{E_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)$$

$$C = \frac{I_R}{2} + \frac{E_R}{2}\sqrt{Y/Z} = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)$$

$$D = \frac{I_R}{2} - \frac{E_R}{2}\sqrt{Y/Z} = \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)$$

TRANSMISSION LINE GENERAL SOLUTION

$$E = A e^{\sqrt{ZY} s} + B e^{-\sqrt{ZY} s}$$

$$= \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{ZY} s} + \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{ZY} s}$$

$$= \frac{E_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{ZY} s} + \left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{ZY} s}\right]$$

$$= \frac{E_R}{2}\left[\frac{Z_R + Z_0}{Z_R} e^{\sqrt{ZY} s} + \left(\frac{Z_R - Z_0}{Z_R}\right) e^{-\sqrt{ZY} s}\right]$$

$$\boxed{E = \frac{E_R}{2 Z_R}(Z_R + Z_0)\left[e^{\sqrt{ZY} s} + \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\sqrt{ZY} s}\right]}$$

$$\boxed{I = \frac{I_R (Z_R + Z_0)}{2 Z_0}\left[e^{\sqrt{ZY} s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} s}\right]}$$

TRANSMISSION LINE GENERAL SOLUTION

$$E = A e^{\sqrt{ZY}s} + B e^{-\sqrt{ZY}s}$$

$$= \frac{E_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{ZY}s}\right] + \frac{E_R}{2}\left[\left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{ZY}s}\right]$$

$$= \frac{E_R}{2}\left[e^{\sqrt{ZY}s} + \frac{Z_0}{Z_R} e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s} - \frac{Z_0}{Z_R} e^{-\sqrt{ZY}s}\right]$$

$$= \frac{E_R}{2}\left[\left(e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}\right) + \frac{Z_0}{Z_R}\left(e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}\right)\right]$$

$$= E_R\left(\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}}{2}\right) + \frac{\overset{I_R Z_R}{E_R}\, Z_0}{Z_R}\left(\frac{e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}}{2}\right)$$

$$\boxed{\begin{aligned} E &= E_R \cosh\sqrt{ZY}s + I_R Z_0 \sinh\sqrt{ZY}s \\ I &= I_R \cosh\sqrt{ZY}s + \frac{E_R}{Z_0} \sinh\sqrt{ZY}s \end{aligned}}$$

$|||^{ly}$

TRANSMISSION LINE GENERAL SOLUTION

Final Solution

After simplifying the above equations we get the final and very useful form of equations for voltage and current at any point on a line, and are solutions to the wave equation.

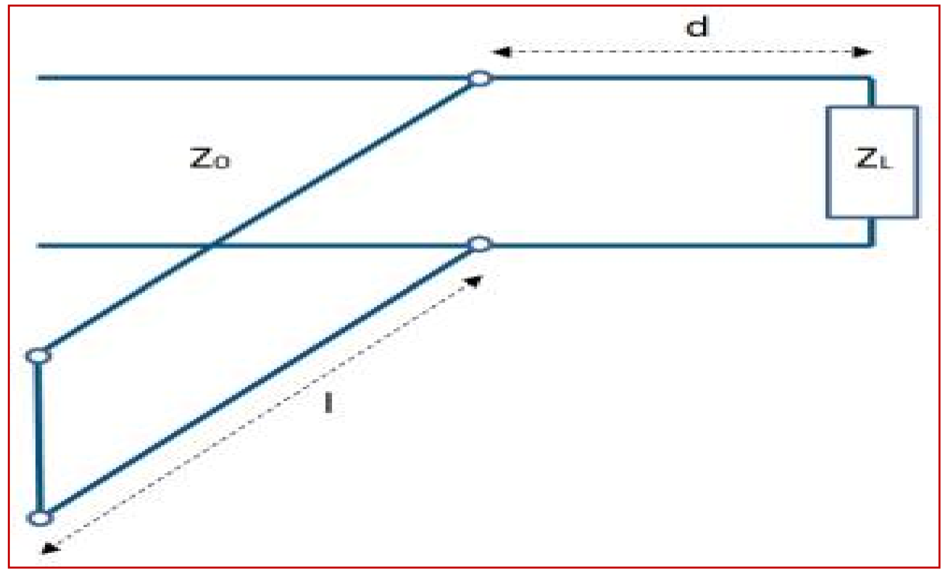
$$E = E_R \cosh\sqrt{ZY}s + I_R Z_0 \sinh\sqrt{ZY}s$$

$$I = I_R \cosh\sqrt{ZY}s + \frac{E_R}{Z_0}\sinh\sqrt{ZY}s$$

The above equations are known as general solution of the transmission line and are the equations of voltage and current at any point on a transmission line.,

APPLICATIONS OF TRANSMISSION LINES

1. They are used to transmit signal i.e. EM Waves from one point to another.
2. They can be used for impedance matching purpose.
3. They can be used as stubs by properly adjusting their lengths.



ASSESSMENT

1. State the difference between lumped and distributed networks.
2. What is the need for transmission line general solution?
3. Mention some of the parameters of the line.
4. What are the assumptions at the receiving end of the line?
5. Give the expressions for transmission line equations.

THANK YOU