Dr. SNS RAJALAKSHMI COLLEGE OF ARTS & SCIENCE (Autonomous)
Coimbatore -641049

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DEPARTMENT OF GRAPHIC & CREATIVE DESIGN AND DATA ANALYTICS

COURSE NAME : COMPUTER SYSTEM ARCHITECTURE (23UCU402)

I YEAR /I SEMESTER

Unit II- Logic Gates

Topic : K-MAP

Karnaugh Maps (K-Maps)

- A visual way to simplify logic expressions
- It gives the most simplified form of the expression

Rules to obtain the most simplified expression

- **Steps to solve expression using K-map-**
- Select K-map according to the number of variables.
- Identify minterms or maxterms as given in problem.
- For SOP put 1's in blocks of K-map respective to the minterms (0's elsewhere).
- For POS put 0's in blocks of K-map respective to the maxterms(1's elsewhere).
- Make rectangular groups containing total terms in power of two like 2,4,8 ..(except 1) and try to cover as many elements as you can in one group.
- From the groups made in step 5 find the product terms and sum them up for SOP form.

Three-Variable K-Maps

$f = \sum(0,4) = \overline{B}\,\overline{C}$

A \ BC	00	01	11	10
0	**1**	0	0	0
1	**1**	0	0	0

$f = \sum(4,5) = A\,\overline{B}$

A \ BC	00	01	11	10
0	0	0	0	0
1	**1**	**1**	0	0

$f = \sum(0,1,4,5) = \overline{B}$

A \ BC	00	01	11	10
0	**1**	**1**	0	0
1	**1**	**1**	0	0

$f = \sum(0,1,2,3) = \overline{A}$

A \ BC	00	01	11	10
0	**1**	**1**	**1**	**1**
1	0	0	0	0

$f = \sum(0,4) = \overline{A}\,C$

A \ BC	00	01	11	10
0	0	**1**	**1**	0
1	0	0	0	0

$f = \sum(4,6) = A\,\overline{C}$

A \ BC	00	01	11	10
0	0	0	0	0
1	**1**	0	0	**1**

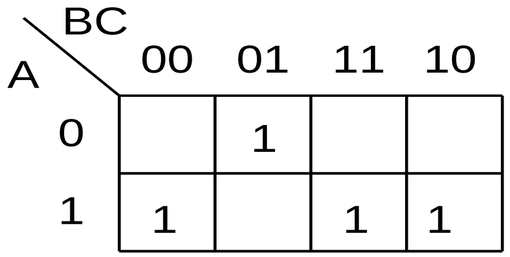
$f = \sum(0,2) = \overline{A}\,\overline{C}$

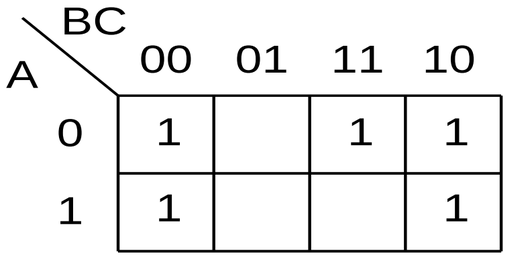
A \ BC	00	01	11	10
0	**1**	0	0	**1**
1	0	0	0	0

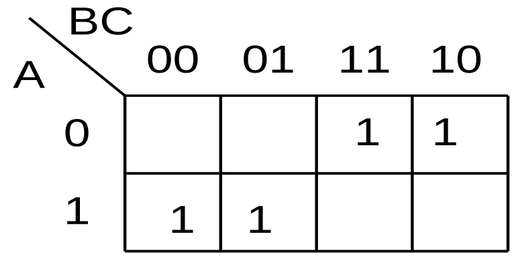
$f = \sum(0,2,4,6) = \overline{C}$

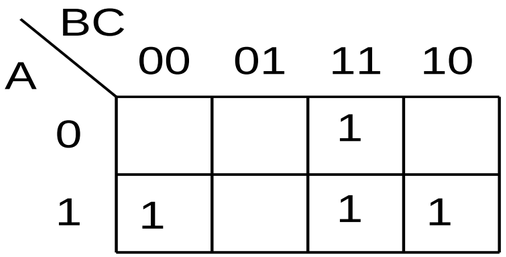
A \ BC	00	01	11	10
0	**1**	0	0	**1**
1	**1**	0	0	**1**

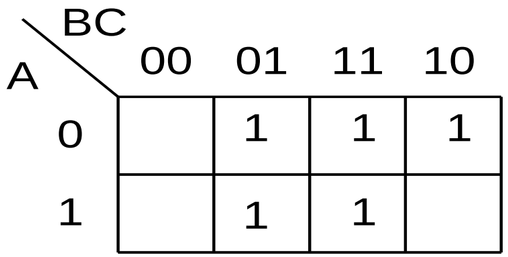
Three-Variable K-Map Examples

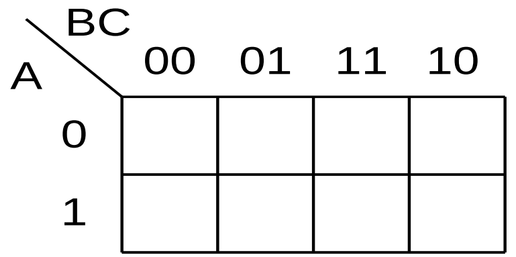












Four-Variable K-Maps

CD / AB	00	01	11	10
00	**1**	0	0	0
01	0	0	0	0
11	0	0	0	0
10	**1**	0	0	0

$$f = \sum(0,8) = \overline{B} \bullet \overline{C} \bullet \overline{D}$$

CD / AB	00	01	11	10
00	0	0	0	0
01	0	**1**	0	0
11	0	**1**	0	0
10	0	0	0	0

$$f = \sum(5,13) = B \bullet \overline{C} \bullet D$$

CD / AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	**1**	**1**	0
10	0	0	0	0

$$f = \sum(13,15) = A \bullet B \bullet D$$

CD / AB	00	01	11	10
00	0	0	0	0
01	**1**	0	0	**1**
11	0	0	0	0
10	0	0	0	0

$$f = \sum(4,6) = \overline{A} \bullet B \bullet \overline{D}$$

CD / AB	00	01	11	10
00	0	0	**1**	**1**
01	0	0	**1**	**1**
11	0	0	0	0
10	0	0	0	0

$$f = \sum(2,3,6,7) = \overline{A} \bullet C$$

CD / AB	00	01	11	10
00	0	0	0	0
01	**1**	0	0	**1**
11	**1**	0	0	**1**
10	0	0	0	0

$$f = \sum(4,6,12,14) = B \bullet \overline{D}$$

CD / AB	00	01	11	10
00	0	0	**1**	**1**
01	0	0	0	0
11	0	0	0	0
10	0	0	**1**	**1**

$$f = \sum(2,3,10,11) = \overline{B} \bullet C$$

CD / AB	00	01	11	10
00	**1**	0	0	**1**
01	0	0	0	0
11	0	0	0	0
10	**1**	0	0	**1**

$$f = \sum(0,2,8,10) = \overline{B} \bullet \overline{D}$$

Four-Variable K-Maps

CD / AB	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$f = \sum(4,5,6,7) = \overline{A} \bullet B$

CD / AB	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$f = \sum(3,7,11,15) = C \bullet D$

CD / AB	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

$f = \sum(0,3,5,6,9,10,12,15)$

$f = A \otimes B \otimes C \otimes D$

CD / AB	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$f = \sum(1,2,4,7,8,11,13,14)$

$f = A \oplus B \oplus C \oplus D$

CD / AB	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$f = \sum(1,3,5,7,9,11,13,15)$

$f = D$

CD / AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$f = \sum(0,2,4,6,8,10,12,14)$

$f = \overline{D}$

CD / AB	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$f = \sum(4,5,6,7,12,13,14,15)$

$f = B$

CD / AB	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$f = \sum(0,1,2,3,8,9,10,11)$

$f = \overline{B}$

Four-Variable K-Maps Examples

AB \ CD	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

AB \ CD	00	01	11	10
00				
01	1	1	1	
11	1	1		1
10	1			

AB \ CD	00	01	11	10
00		1	1	
01	1	1	1	1
11	1		1	1
10			1	

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

Four-Variable K-Maps Examples

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

Four-Variable K-Maps Examples

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

Design of combinational digital circuits

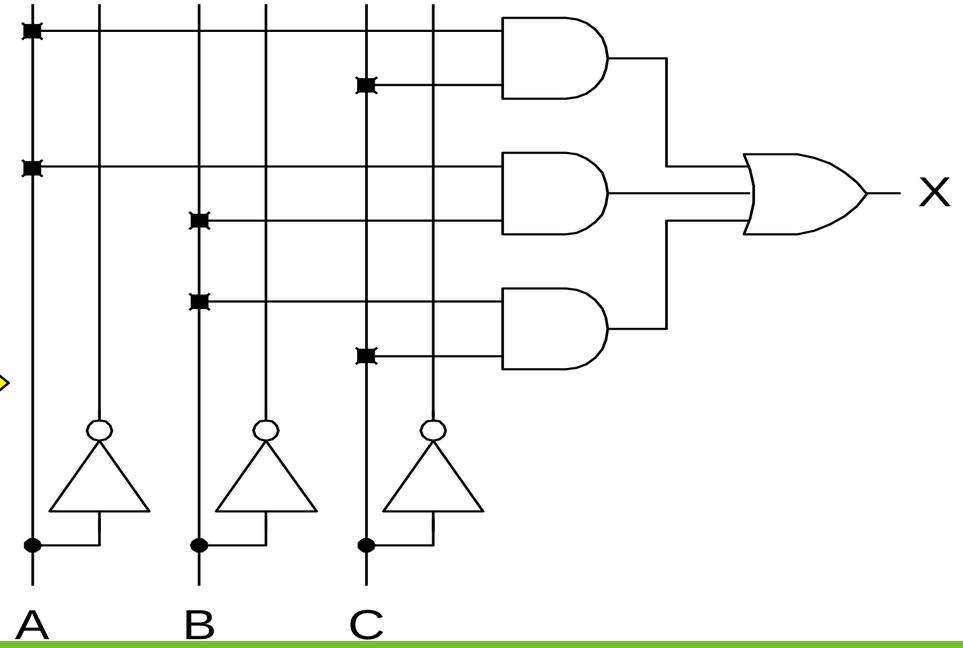
- Steps to design a combinational digital circuit:
  - From the problem statement derive the truth table
  - From the truth table derive the unsimplified logic expression
  - Simplify the logic expression
  - From the simplified expression draw the logic circuit
- Example: Design a 3-input (A,B,C) digital circuit that will give at its output (X) a logic 1 only if the binary number formed at the input has more ones than zeros.

	Inputs A	B	C	Output X
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$$X = \sum(3, 5, 6, 7)$$

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$X = AC + AB + BC$$



Design of combinational digital circuits (Cont.)

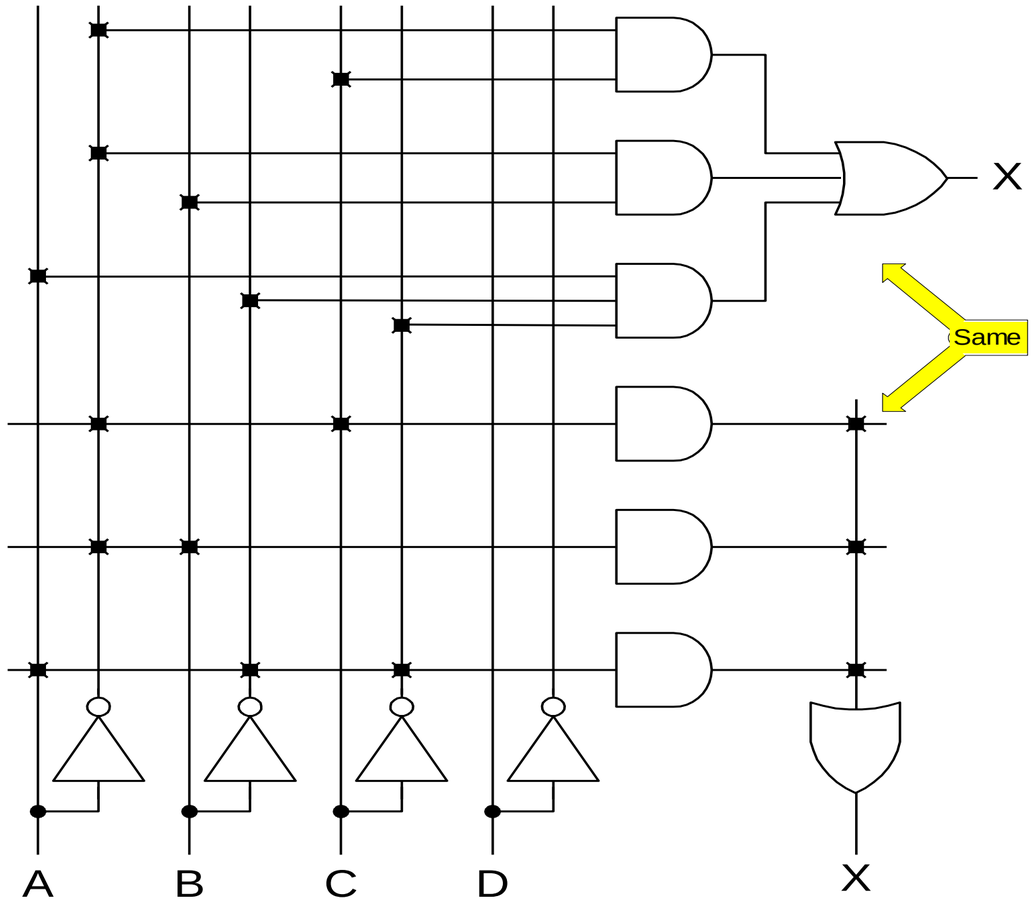
- Example: Design a 4-input (A,B,C,D) digital circuit that will give at its output (X) a logic 1 only if the binary number formed at the input is between 2 and 9 (including).

	Inputs				Output
	A	B	C	D	X
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

$$X = \sum(2,3,4,5,6,7,8,9)$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

$$X = \overline{A}C + \overline{A}B + A\overline{B}\,\overline{C}$$



Design of combinational digital circuits (Example)

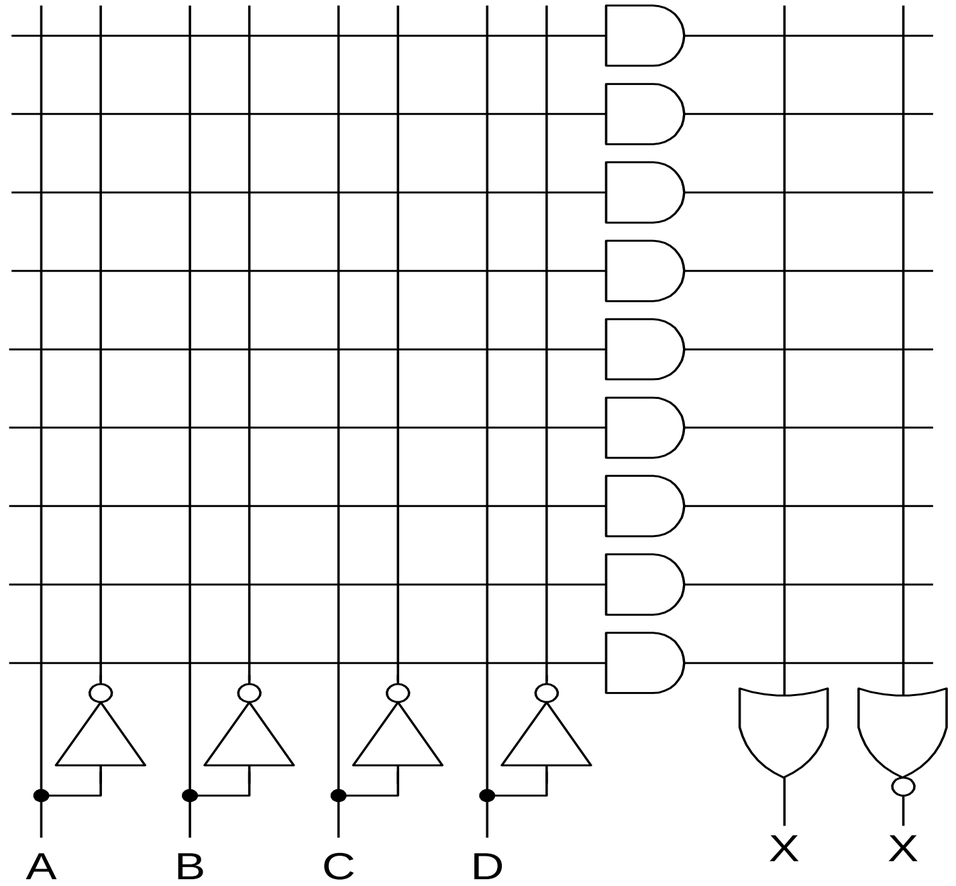
- Example: Design a 4-input (A,B,C,D) digital circuit that will give at its output (X) a logic 1 only if there more ones than zeros in the binary number formed at the input.

	Inputs				Output		
	A	B	C	D			
0	0	0	0	0			
1	0	0	0	1			
2	0	0	1	0			
3	0	0	1	1			
4	0	1	0	0			
5	0	1	0	1			
6	0	1	1	0			
7	0	1	1	1			
8	1	0	0	0			
9	1	0	0	1			
10	1	0	1	0			
11	1	0	1	1			
12	1	1	0	0			
13	1	1	0	1			
14	1	1	1	0			
15	1	1	1	1			

X =

AB \ CD	00	01	11	10
00				
01				
11				
10				

X =



References

1.M.Morris Mano, “Computer System Architecture” 3rd Edition, Prentice Hall of India ,2000, ISBN-10: 0131663631

2. V.K. Puri, —DIGITAL ELECTRONICS CIRCUITS AND SYSTEMS” McGraw Hill Education (1 July 2017). ISBN-10: 9780074633175 , ISBN-13: 978-0074633175

3.William Stallings, “Computer Organization and Architecture, Designing for Performance” PHI/ Pearson Education North Asia Ltd., 10th Edition 2016, ISBN 978-0- 13-410161-3 — ISBN 0-13-410161-8.

Thank You