

## Department of Mathematics

### 23MAT103 – DIFFERENTIAL EQUATIONS AND TRANSFORMS

I B.E./ B.Tech. / I SEMESTER

#### UNIT I : Vector Calculus

**Topic : Divergence and Curl of a Vector field and  
Solenoidal and irrotational of a vector**



# History



**Joseph-Louis Lagrange (1762)**

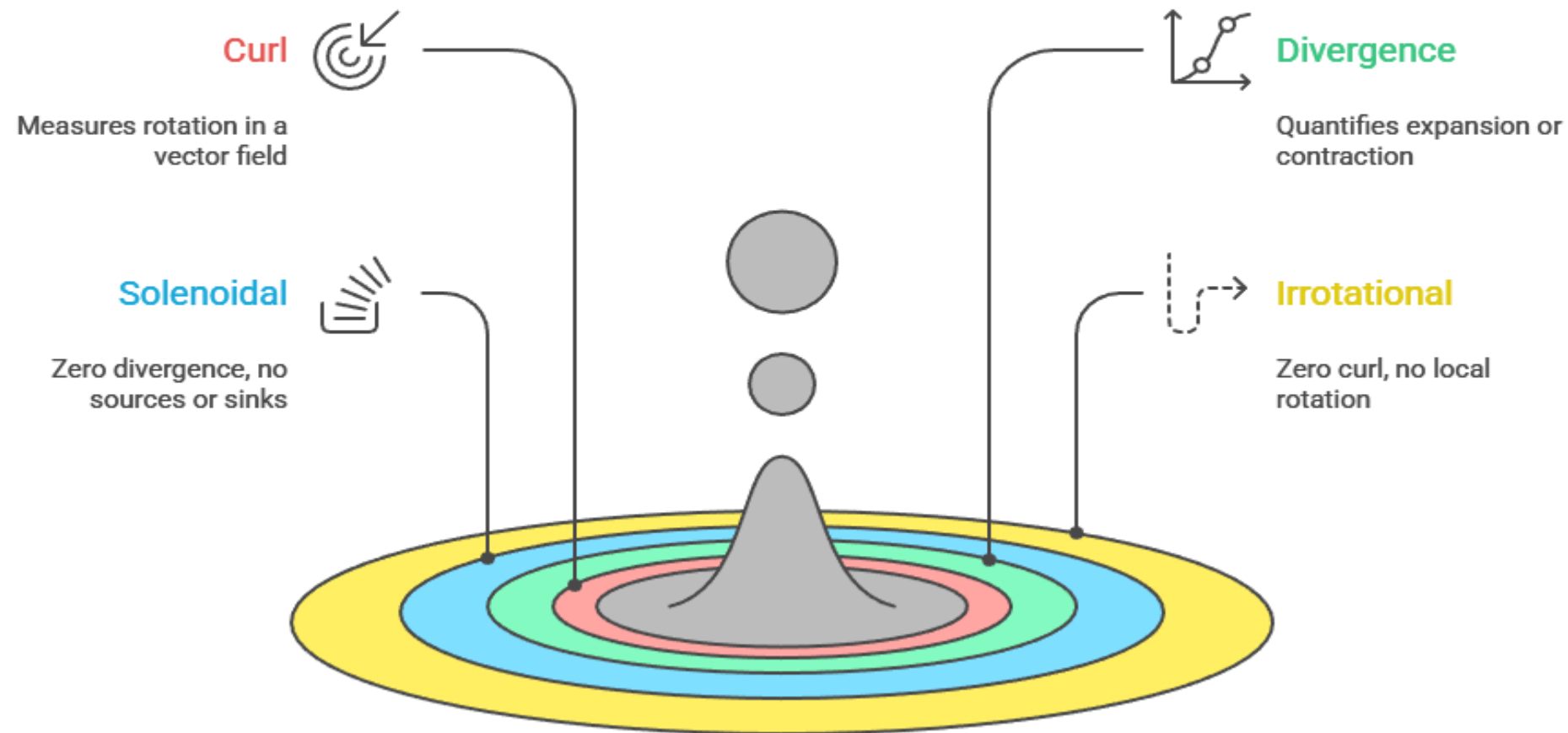


**James Clerk Maxwell (1839)**



**André-Marie Ampère in 1823**

## Vector Field Analysis

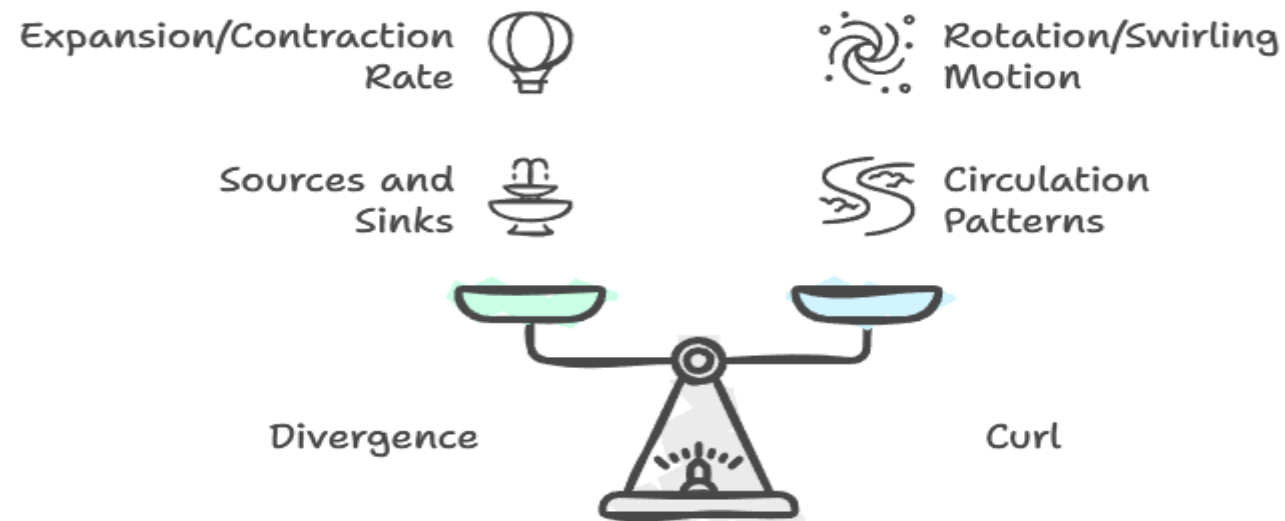


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# Divergence and Curl of a Vector field

Curl of a vector field is the vector obtained by taking the cross product of the del operator with the vector field.

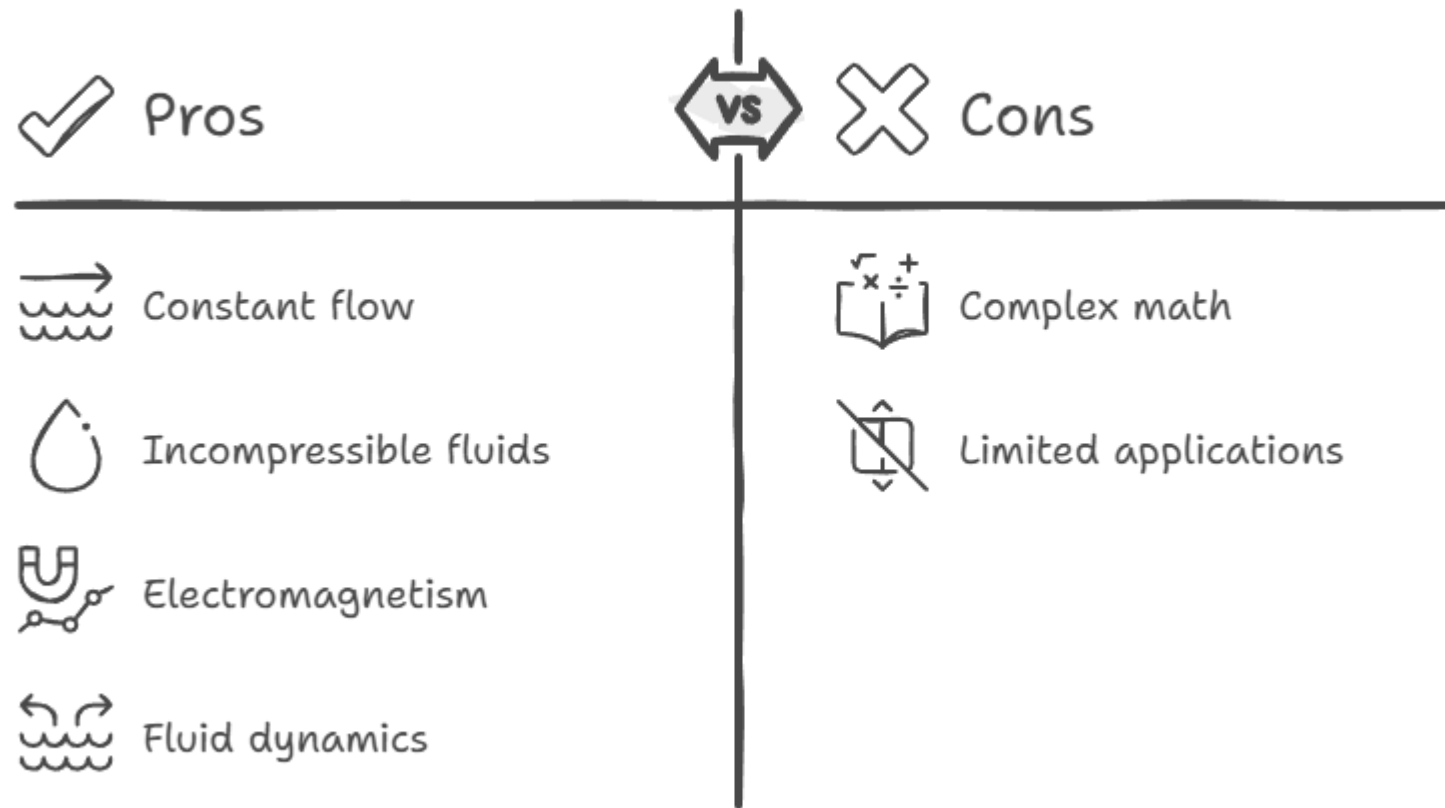
## Divergence and Curl: Understanding Vector Field Dynamics



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# Solenoidal Vector Field

## Solenoidal vector fields



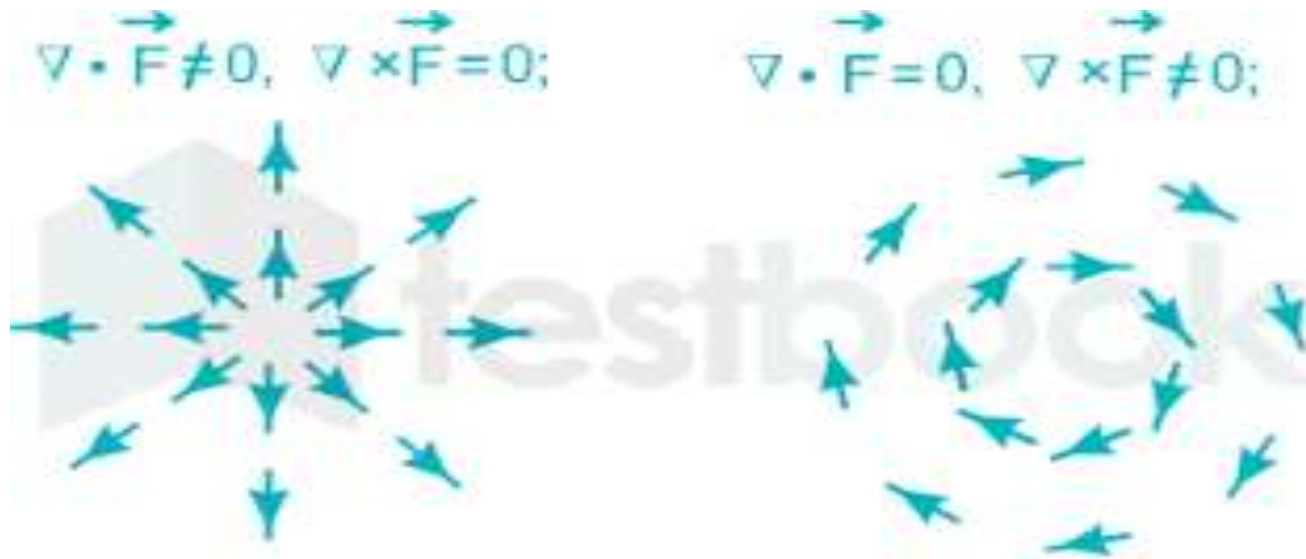
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# Applications

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A **vector field** is said to be **solenoidal** if its **divergence is zero everywhere**.

$$\nabla \cdot \vec{F} = 0$$



# Difference between Solenoidal and Curl

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## Solenoidal Vector Field

A vector field is **solenoidal** if its **divergence is zero**.

Mathematical condition:  $\nabla \cdot \vec{F} = 0$

Indicates **no source or sink** in the field.

Field lines form **closed loops**.

Related to **divergence**.

Example: Magnetic field is solenoidal.

## Curl of a Vector Field

**Curl** measures the **rotation or circulation** of a vector field.

Mathematical expression:  $\nabla \times \vec{F}$

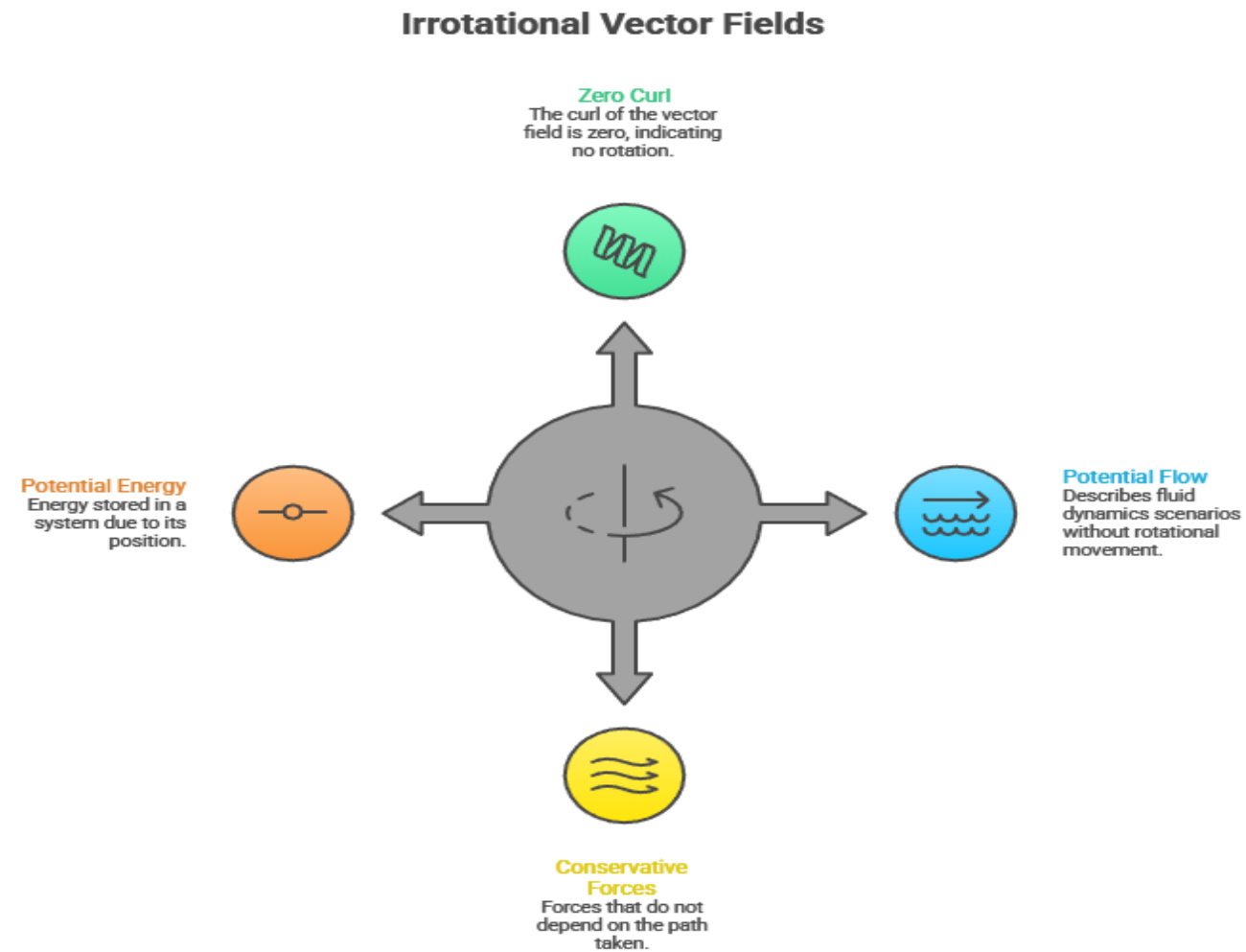
Indicates the **tendency to rotate** about a point.

Curl gives a **vector quantity** showing axis and magnitude of rotation.

Related to **rotational nature** of the field.

Example: Velocity field of a rotating fluid has non-zero curl.

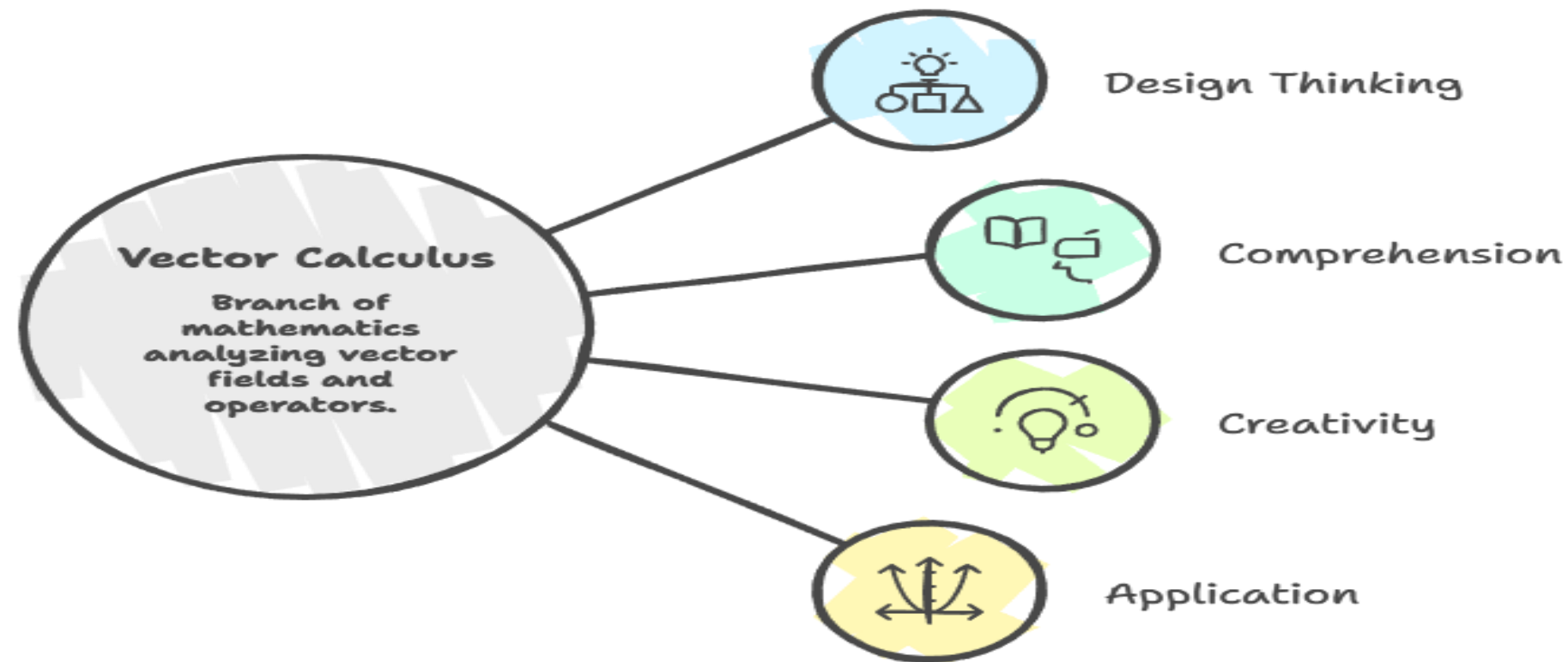
# Irrotational of a vector



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# Vector Calculus in Design Thinking

## Unveiling the Dimensions of Vector Calculus

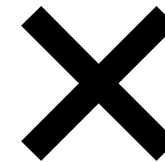
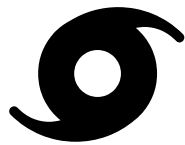


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# Activity

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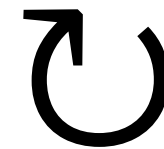
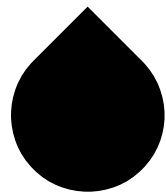
Identify the word from the pictures given



# Activity

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Identify the word from the pictures given



# References

- <https://mathinsight.org/divergence>
- <https://www.geeksforgeeks.org/engineering-mathematics/vector-calculus/>
- [https://mathinsight.org/vector\\_calculus](https://mathinsight.org/vector_calculus)

