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Coimbatore- 49

DEPARTMENT OF MATHEMATICS

21UCR304: BUSINESS CALCULUS AND FINANCIAL COMPUTATION

Contingent Payments

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What is a Contingent Payment?

A contingent payment is a payment whose amount, or the very fact of its being made, depends on the occurrence of some uncertain event. In finance and actuarial science, these payments combine probability theory with the time value of money.

📌 **Key Point:**

Unlike fixed payments, contingent payments may or may not be received. Their valuation requires knowledge of probabilities and discounting.

Probability — Core Concepts

Probability measures the likelihood of an event occurring, ranging from 0 (impossible) to 1 (certain).

Sample Space (S): The set of all possible outcomes of an experiment.

Event (E): A subset of the sample space; one or more outcomes.

P(E) = (Number of favourable outcomes) / (Total number of outcomes) — Classical definition.

Complement Rule: $P(E') = 1 - P(E)$, where E' is the event NOT happening.

Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Rule (independent events): $P(A \cap B) = P(A) \times P(B)$

Conditional Probability: $P(A|B) = P(A \cap B) / P(B)$

Mathematical Expectation (Expected Value)

- Mathematical Expectation (E[X]) is the weighted average of all possible values a random variable can take.
- Each value is weighted by its probability of occurrence.
- For a discrete random variable X with values $x_1, x_2, \ldots, x_n$ and corresponding probabilities $p_1, p_2, \ldots, p_n$:
- The sum of all probabilities must equal 1: $\Sigma p_i = 1$
- Example: A game pays ₹500 with prob 0.4, ₹200 with prob 0.35, ₹0 with prob 0.25.
- $E[X] = 500 \times 0.4 + 200 \times 0.35 + 0 \times 0.25 = 200 + 70 + 0 =$ ₹270
- Decision Rule: Accept a bet/investment if $E[X] > 0$ (positive expected gain).

FORMULA

$$E[X] = \Sigma x_i \cdot p_i$$

$$i = 1, 2, \ldots, n$$

Properties of Mathematical Expectation

Linearity: $E[aX + b] = a \cdot E[X] + b$, where a and b are constants.

$E[X + Y] = E[X] + E[Y]$ (always true, regardless of independence).

$E[X \cdot Y] = E[X] \cdot E[Y]$ only when X and Y are independent.

Variance: $Var(X) = E[X^2] - (E[X])^2$ → measures spread around the mean.

Standard Deviation: $\sigma = \sqrt{Var(X)}$ → same units as X.

If $X \geq 0$ always, then $E[X] \geq 0$.

$E[\text{constant}] = \text{constant}$.

Contingent Payments with Time Value of Money

- When a payment is both uncertain (contingent) and future (time-valued), we find its Present Value of Expected Payment.
- Combine probability of receiving a payment with discounting to present value.
- PV of a contingent payment = Probability × (Payment Amount) × Discount Factor
- Discount Factor: $v = 1/(1+i)^n$, where i = interest rate, n = years.
- Example: ₹10,000 due in 3 years with probability 0.7; i = 6%
- $PV = 0.7 \times 10{,}000 \times (1/1.06^3) = 0.7 \times 10{,}000 \times 0.8396 =$ ₹5,877
- This concept is the foundation of life insurance and annuity pricing.

FORMULA

$$PV = p \times C \times v^n$$
$$v = 1/(1+i)^n$$

THANK YOU