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DEPARTMENT OF MATHEMATICS

25UCU305: DISCRETE MATHEMATICS WITH PROBABILITY AND HYPOTHESIS TESTING

COMPOSITION OF FUNCTIONS

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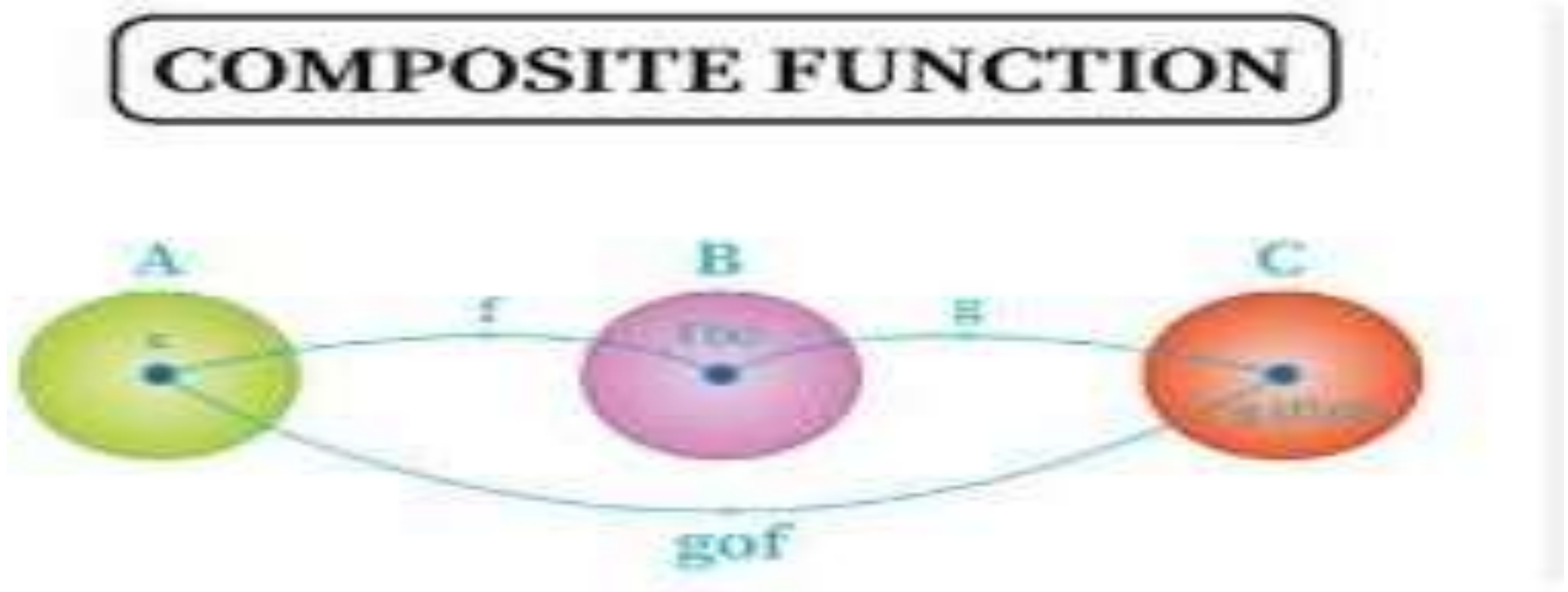
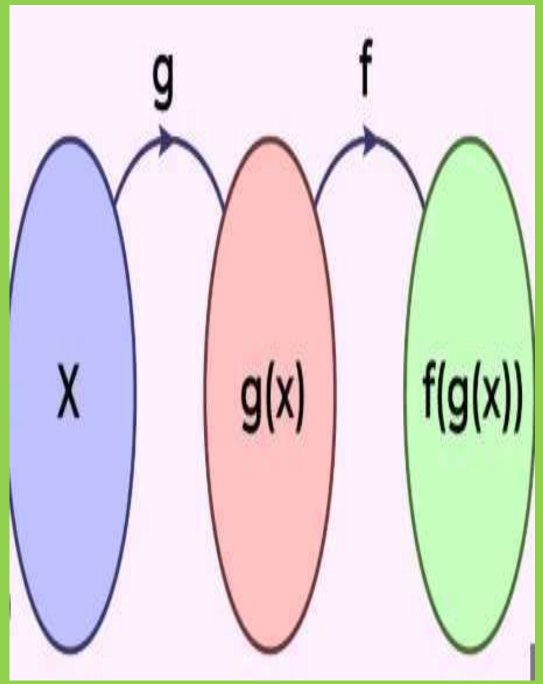
Composition of Functions

Composition of functions combines two functions to form a new function.

If f maps A → B and g maps B → C, then g ∘ f maps A → C.

It shows how outputs of one function become inputs of another.



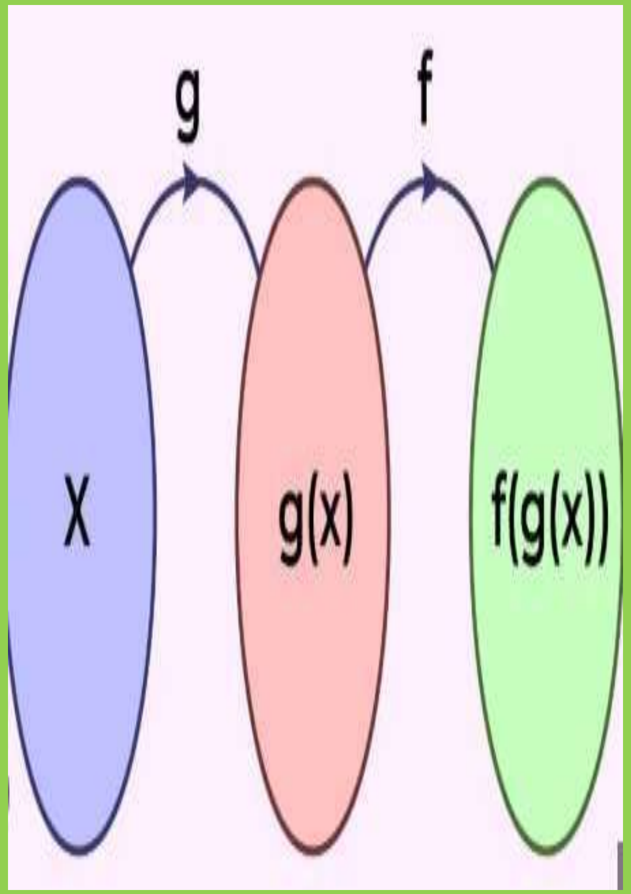


Definition :

Let f: A → B, g: B → C
The composition of g with f is:

$$(g \circ f)(x) = g(f(x))$$

First apply f, then apply g.
Domain of g ∘ f = Domain of f.



Process of Composition

To find g∘f(x):

1. Take **x** from A
2. Apply **f** to get f(x)
3. Take f(x) and apply **g**
4. Final output is g(f(x))

Example 1

Let

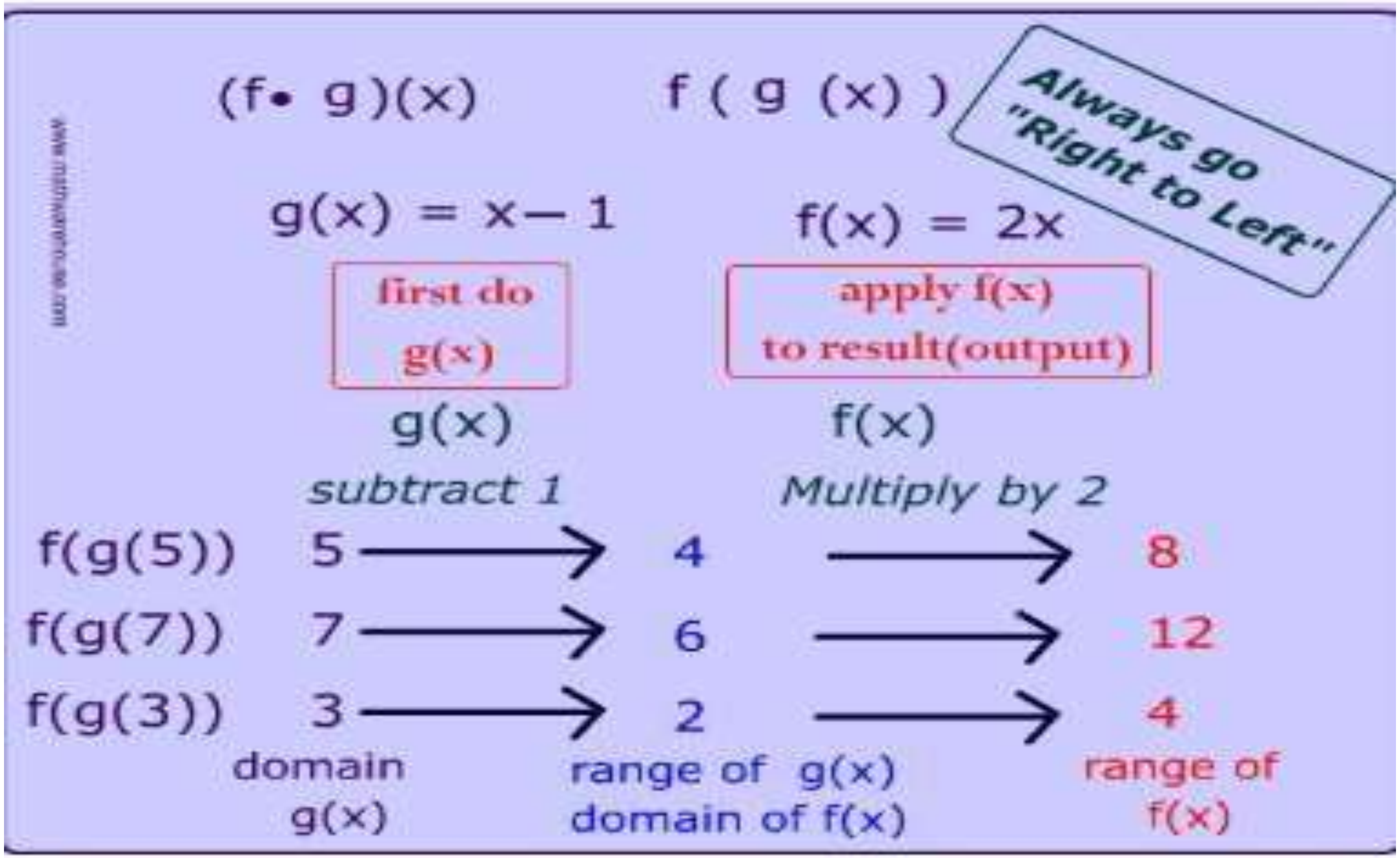
$f(x) = 2x + 1$

$g(x) = x^2$

Compute $(g \circ f)(x)$:

$$(g \circ f)(x) = g(2x + 1) = (2x + 1)^2$$

Composition of Functions



Example 2

Let $f(x) = x - 3$, $g(x) = 5x$

Compute $(f \circ g)(x)$:

$$(f \circ g)(x) = f(5x) = 5x - 3$$

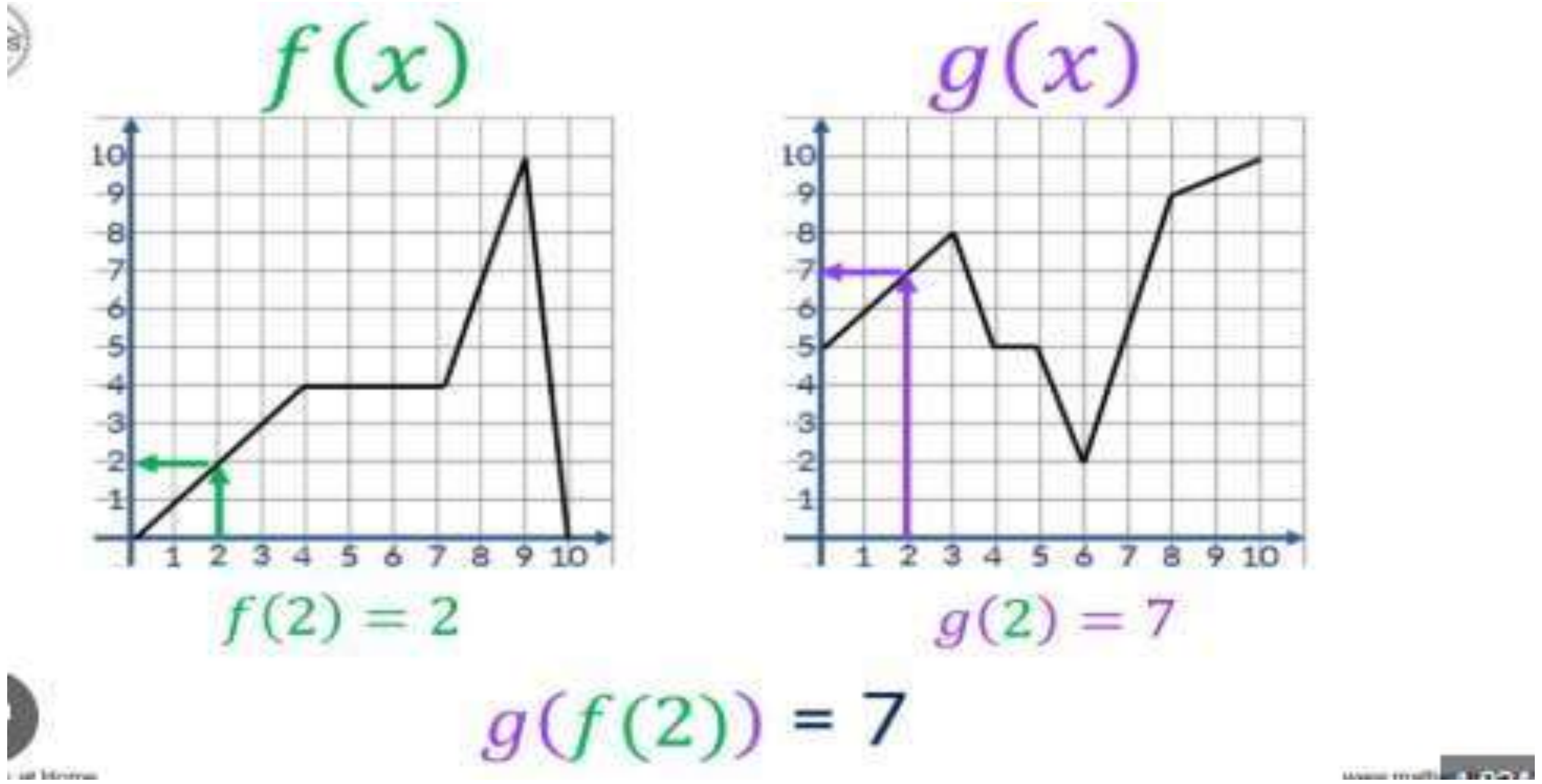
Note: $g \circ f$ and $f \circ g$ are usually not equal.

Example 3 (Functional Mapping)

Set A = {1, 2, 3}
Set B = {2, 4, 6}
Set C = {4, 8, 12}
f = {(1,2), (2,4), (3,6)}
g = {(2,4), (4,8), (6,12)}
Composition g ∘ f:
1 → 2 → 4
2 → 4 → 8
3 → 6 → 12
So, g ∘ f = {(1,4), (2,8), (3,12)}

Graphical Interpretation

- Functions can be visualized as arrows connecting sets.
- Composition chains these arrows into a single mapping from the first set to the last.
- Useful in transformations, geometry, and computer science.



Properties of Composition

Associative:

$$h \circ (g \circ f) = (h \circ g) \circ f$$

Not commutative:

$$g \circ f \neq f \circ g$$

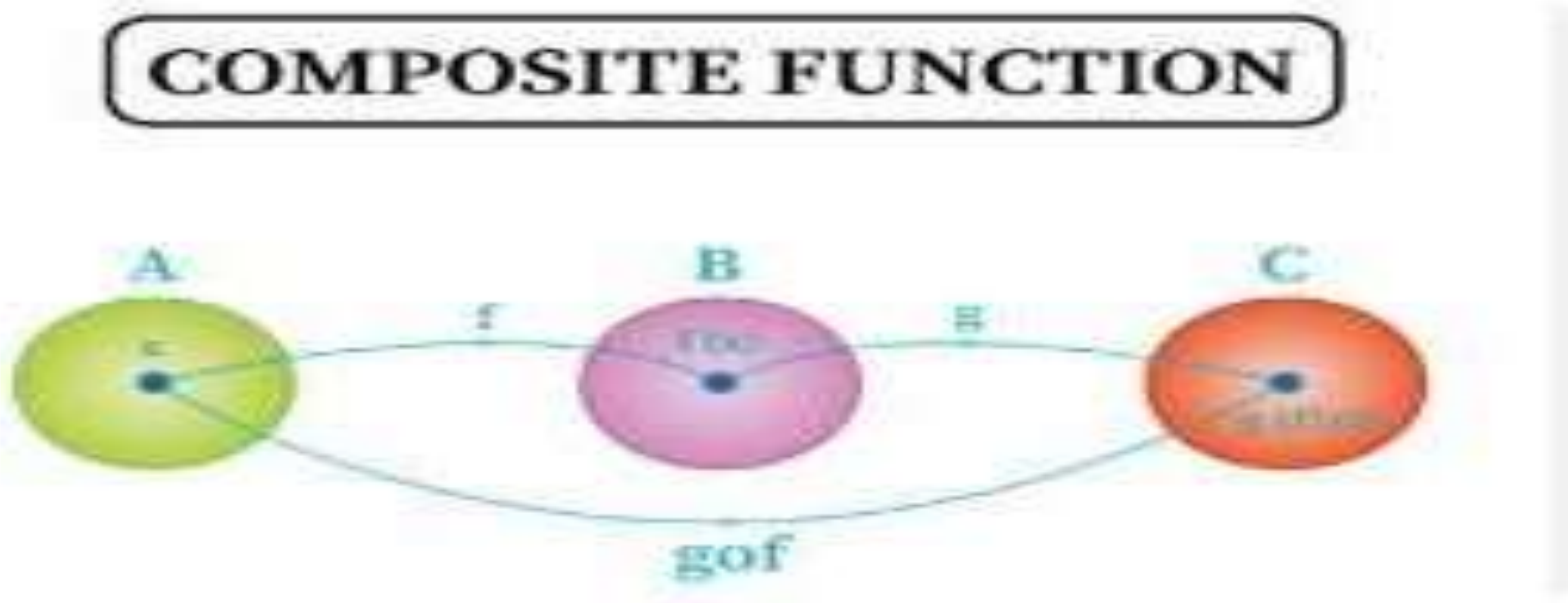
Identity function acts as a neutral element.

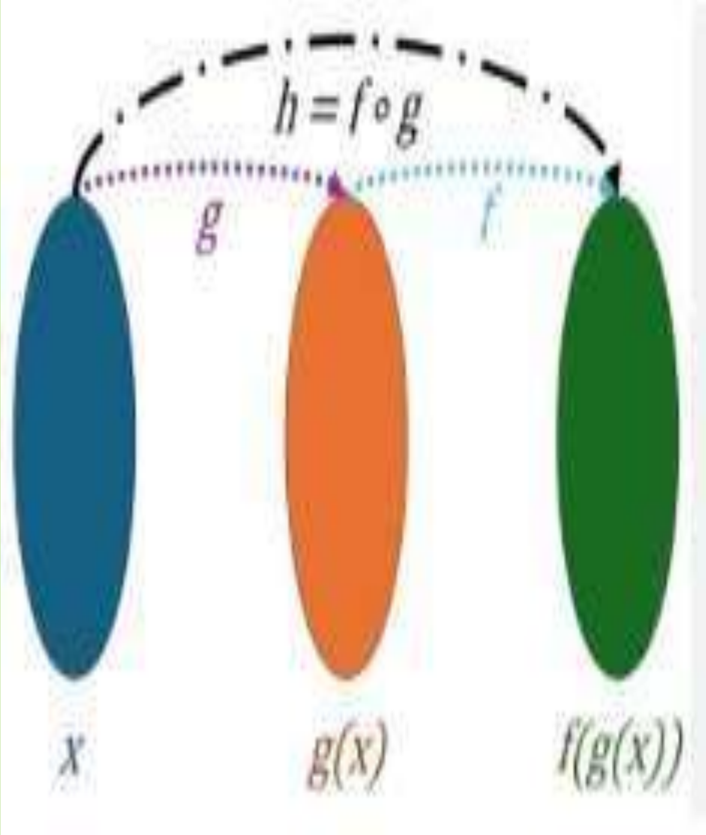
Application Areas

- Computer algorithms
- Data processing pipelines
- Mathematical modeling
- Geometric transformations
- Input–output systems
- Machine learning functions

Composition of Functions

If f: A → B and g: B → C,
Then the composition is: $g \circ f(x) = g(f(x))$
Connects two functions into one process.





Composition of Functions

KEY CONCEPT *For Your Notebook*

Operations on Functions

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$h(x) = f(x) + g(x)$	$h(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{5x}{x + 2}$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

Summary

- Composition combines two functions into one.
- $(g \circ f)(x) = g(f(x))$
- Order of composition is important.
- Widely applicable in mathematics and computer science.

Thank You