Subject: SUPPORTIVE / ALLIED MATHEMATICS - 2: DISCRETE MATHEMATICS WITH PROBABILITY AND HYPOTHESIS TESTING **Code:** 25UCU305

QUESTION AND ANSWER
UNIT: 1

1. Define Binary relations and Reflexive Relations

2. Let f:R→R and g:R→R be a function defined on the x set of real numbers. Find $f \circ g$and $g \circ f$ where $f(x) = x^2 - 2 \ \ and \ \ g(x) = x + 4$

3. Define Equivalence relations and give example for it.

4. Let f:R→R and g:R→R be a function defined on the x set of real numbers. Find $f \circ g$and $g \circ f$ where $f(x) = 4x^2 - 1 \ \ and \ \ g(x) = 1 + x$

5. Let f:R→R and g:R→R be a function defined on the x set of real numbers. Find $f \circ g$and $g \circ f$ where $f(x) = x - 6 \ \ and \ \ g(x) = x^3$

6. Let $f(x) = x^2 - 5x + 6$ find $f(1), f(2), f(0)$

7. Express the relation R from A= {3,5,7,9,11} into B= {2,6,8,10} as a set of ordered pairs defined by the sentence "R: is one less than"

8. Find the Cartesian Product of AxB, BxA for the sets A={1,3,4} and B={5,6}

9. Check whether the following functions are one-to-one and onto. f:N→Ndefined byf(n)=n+2. (Draw an arrow diagram)

10. Find the value of k such that $f \circ g$=$g \circ f$ where $f(x) = 3x + 2 \ \ and \ \ g(x) = 6x - K$

11. Find the value of k such that $f \circ g$=$g \circ f$ where $f(x) = 2x - K \ \ and \ \ g(x) = 4x + 5$

12. Let $f(x) = x^2 - 1 Find (i) f \circ f (ii) f \circ f \circ f$

13. A function f is defined by f(x)=3-2x. find x such that $f(x^2) = (f(x))^2$.

14. If A= {-2,-1,0,1,2} and f:A→B is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

15. Let S={1,2,3} and R={(1,1),(1,2),(2,2),(1,3),(3,1)}. i)Is R reflexive? If not, state the reason and write the minimum set of ordered pairs to be included to R so as to make it reflexive. ii)Is R symmetric? If not, state the reason, write minimum number of ordered pairs to be included to R so as to make it symmetric and write minimum number of ordered pairs to be deleted from R so as to make it symmetric.

16. If f(x)=2x+3, g(x)=1-2x and h(x)=3x. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

17. Define (i) Binary Relation (ii) Inverse Relation (iii) Equivalence relation

18. Let A={2,5,6}and let r be the relation {(2,2),(2,5),(5,6),(6,6)} on A, Find the adjacency matrix r and r^2.

19. If A={5,6}, B={4,5,6}, C={5,6,7} then show that $A \times A = (B \times B) \cap (C \times C)$

20. Let A = {a, b, c}. What is the equivalence relation of smallest cardinality on A? What is theequivalence relation of largest cardinality on A?

21. Let A={1,2,3,4}, B={1,5,9,11,15,16} and f={(1,5),(2,9),(3,1),(4,5),(2,11)} Are the following true i)f is a relation from A to B ii)f is a function from A to B? Justify your answer.

22. Let f be a function defined by f(x)=3x+2 then (i) Find the images of 1, 2, 3 (ii) find the pre-images of 29,53.

23. Let the functions f and g on the real numbers be defined by $f(x) = x^2 + 2x - 3, g(x) = 3x - 4$ Find $g \circ f and f \circ g$

24. In the set Z of integers, define mRn if m – n is divisible by 7. Prove that R is an equivalencerelation.

25. The function for exchanging American dollars for Singapore Dollar on a given day isf(x)=1.3x where x represents the number of American dollars.On the same day the function forexchanging Singapore Dollar to Indian Rupee isg(y)=69.7y, where y represents the numberof Singapore dollars. Write a function which will give the exchange rate of American dollars interms of Indian rupee.

26. The owner of a small restaurant can prepare a particular meal at a cost of Rupees 100. He estimatesthat if the menu price of the meal is x rupees, then the number of customers who will order thatmeal at that price in an evening is given by the functionD(x)=250-x. Express his day revenue,total cost and profit on this meal as functions of x.

27. A simple cipher takes a number and codes it, using the functionf(x)=4x+5.Find the inverse ofthis function, determine whether the inverse is also a function

28. Let f be a function from R to R defined by f(x)=3x-5. Find the values of a and b given that (a,4) and (1,b) belong to f

29. Explain the Types of relations with Examples

30. Let x={1,2,3} and f,g,h and s be function from x to x given by f: {(1,2),(2,3),(3,1)}, g:{(1,2),(2,1),(3,3)}, h:{(1,1),(2,2),(3,1)} and s={(1,1),(2,2),(3,3)} find$f \circ g$, \($g \circ f$, $f \circ h \circ g$, $g \circ s$, $s \circ s$, $f \circ s$.

31. Let $f(x) = x - 1,\ g(x) = 3x + 1\ and\ h(x) = x^2$ then prove that $(f \circ g) \circ h = f \circ (g \circ h)$

32. If X={1,2,3,4,5,6,7} and R= $(x, y)|x - yisdivisibleby3$.Show that R is an equivalence relation.

33. If f: R→R is defined by f(x)=2x-3prove that f is a bijection and find itsinverse.

34. Let A={0,1,2,3}. Construct relations on A of the following types: i) not reflexive, not symmetric, not transitive. ii)not reflexive, not symmetric, transitive. iii) not reflexive, symmetric, not transitive. Iv)not reflexive, symmetric, transitive.

35. Let A={0,1,2,3}. Construct relations on A of the following types: i)reflexive, not symmetric, not transitive. ii) reflexive, not symmetric, transitive. Iii)reflexive, symmetric, not transitive. Iv)reflexive, symmetric, transitive.

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QUESTION AND ANSWER
UNIT: 2

1. Define Conjunction and disjunction with example
2. Verify Whether $(P \vee Q) \vee (P \wedge Q)$ is contradiction or tautology
3. Define tautology and contradiction with example
4. Indicate which one are tautologies or contradictions.(i) $(P \wedge Q) \leftrightarrow P$(ii) $P \rightarrow (P \vee Q)$
5. Show that it is tautology $(P \wedge Q) \longrightarrow (P \wedge Q)$
6. Construct the Truth Table for the $P \wedge (P \rightarrow Q)$
7. Construct the Truth Table for the $Q \wedge \sim (P \rightarrow Q)$
8. Construct the Truth Table for the$\sim P \wedge \sim Q$
9. Construct the Truth table for $\sim (P \wedge \sim Q)$
10. Write each of the following sentences in symbolic from using statement variables P and Q. (i) 19 is not a prime number and all the angles of a triangle are equal. (ii) 19 is a prime number or all the angles of a triangle are not equal. (iii) 19 is not a prime number
11. Prove $P \rightarrow (q \rightarrow r) \equiv (P \wedge q) \rightarrow r$ without using truth table.
12. Check whether the statement $P \rightarrow (Q \rightarrow P)$ is a tautology or a contradiction.
13. Let P: Mercury is a planet and India is an Island be any two simple statement. Give verbal sentence describing each of the following. (i) $\sim PQ$. (ii) $P \wedge \sim Q$. iii) $\sim P \wedge Q$.
14. Write the statements in words corresponding to $\sim P, P \wedge Q, P \vee Q and P \wedge \sim Q$ where P is 'It iscold' and Q is 'It is raining.'
15. Prove the following Equivalences (i) $\sim (P \longrightarrow Q) \Leftrightarrow P \wedge \sim Q$ (ii) $\sim (P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$
16. Define conditional and Biconditional statements with example
17. Construct the truth table $(\sim P \vee Q) \wedge (\sim Q \vee P)$
18. Explain the five Basic Connectives with example
19. Obtain PCNF for$(Q \rightarrow P) \wedge (\sim P \wedge Q)$
20. Show that $P \Leftrightarrow P \vee (P \wedge Q)$
21. Show that $\sim (P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$
22. Show that $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a Tautology
23. Show that $P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$
24. Construct the truth table for the statement $(\sim P \rightarrow R) \wedge (P \longleftrightarrow Q)$
25. Construct the truth table for the statement $(P \vee Q) \vee \sim Q$

26. Show that $P \rightarrow QandQ \rightarrow P$ are not equivalent.

27. Using the truth table check whether the statements $\sim P \vee Q, \sim P \wedge Qand \sim P$ are logically equivalent.

28. Prove that $Q \rightarrow P \equiv \sim P \rightarrow \sim Q$

29. Show that P is equivalent to $\sim\sim P, P \wedge P, P \vee P, P \wedge (P \vee Q,), (P \wedge Q) \vee (P\wedge \sim Q)$ using truth table.

30. Show that $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

31. Prove that $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

32. Determine the Truth table $(P \wedge Q) \vee (\sim P \wedge Q) \vee (P\wedge \sim Q) \vee (\sim P\wedge \sim Q)$

33. Prove that $P \rightarrow \sim Q \vee R \equiv \sim P\vee \sim Q \vee R$ using truth table

34. Show that $P \longleftrightarrow Q \equiv (\sim P \vee Q) \wedge (\sim Q \vee P)$

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QUESTION AND ANSWER
UNIT: 3

1. Verify whether the following compound propositions are Tautology or Contradiction or Contingency.
$(P \to Q) \wedge (Q \to R) \to (P \to R)$.

2. If two coins are tossed simultaneously, then find the probability of getting (i) One head and one tail (ii) at least one head.

3. When a pair of balanced dice is rolled, what are the probabilities of getting the sum (i) 7 (ii) 7 or 11 (iii) 11 or 12

4. Five mangoes and 4 apples are in a box. If two fruits are chosen at random, find the probability that (i) one is mango and the other is an apple (ii) both are of the same variety.

5. What is the chance that (i) non-leap year (ii) leap year should have fifty-three Sundays?

6. If A and B are mutually exclusive events P(A)=3/8 and P(B)=1/8 then find (i) $P(\underline{A})(ii)P(A \cup B)(iii)P(\underline{A} \cup \underline{B})$

7. If A and B are two independent events such that P(A∪B)=0.6,P(A)=0.2.find P(B).

8. If P(A)=0.5,P(B)=0.8 and P(B/A)=0.8 find P(A/B).Given that P(A) = 0.35, P(B) =0.73, and P(A∩B)=0.14. find (i) $P(A \cup B)$,(ii) $P(\underline{A} \cap B)$ (iii) $P(A \cap \underline{B})$

9. A die is rolled. If it shown an odd number, then find the probability of getting 5.

10. A number is selected from the set {1,2,3,...20}. The probability that the selected number is divisible by 3 or 4.

11. Two units are chosen from a lot containing twelve units, of which four are defective. Find the probability that at least one of the units is defective.

12. If A and B are two independent events such thatP(A)=0.5 and P(A∪B)=0.8,Find P(B)

13. An urn contains 10 white and 5 black balls. While another urn contains 3 white and 7 black balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are white.

14. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is chosen is a prime or multiple of 8?

15. A Cricket club has 16 members of whom only 5 can bowl. What is the probability that in a team of 11 members at least 3 bowlers are selected?

16. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it willget both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards?

17. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane in the first, second, third, and fourth shot are respectively 0.2, 0.4, 0.2 and 0.1. Find the probability that the gun hits the plane.

18. X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

19. The probability that a car being filled with petrol will also need an oil change is 0.30; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.15. (i) If the oil had to be changed, what is the probability that a new oil filter is needed? (ii) If a new oil filter is needed, what is the probability that the oil has to be changed?

20. There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

21. A firm manufactures PVC pipes in three plants viz., X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production, (i) find the probability that the selected pipe is a defective one. (ii) If the selected pipe is a defective, then what is the probability that it was produced by plant Y?

22. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television.

23. A factory has two machines I and II. Machine I and II produce 30% and 70% of items respectively. Further 3% of items produced by Machine I are defective and 4% of items produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.

24. A consulting firm rents car from three agencies such that 20% from agency X, 30% from agency Y and 50% from agency Z. If 90% of the cars from X, 80% of cars from Y and 95% of the cars from Z are in good conditions (i) What is the probability that the firm will get a car in good condition? Also (ii) If a car is in good condition, what is probability that it has come from agency Y?

25. Two thirds of students in a class are boys and rest girls. It is known that the probability of a girl getting a first class is 0.75 and that of a boy is 0.70. Find the probability that a student chosen at random will get first class marks.

26. A husband and wife appear in an interview for two vacancies in the same post. The probability of husbands' selection is 1/6 and that of wife's selection is 1/5. What is the probability that (i) both of them will be selected (ii) only one of them will be selected (iii) none of them will be selected?

27. The probability that a new ship will get an award for its design is 0.25, the probability that it will get an award for the efficient use of materials is 0.35, and that it will get both awards is 0.15. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards?

28. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then find the probability of neither A nor B occurring.

29. The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25, and the probability that she will get both is 0.07. Find the probability that (i) she will get at least one of the two jobs (ii) she will get only one of the two jobs.

30. A problem in Mathematics is given to three students whose chances of solving it are 1/3,1/4,1/5 (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?

31. A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?

32. The chances of X, Y and Z becoming managers of a certain company are 4: 2: 3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?

33. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N?

34. A coin is tossed twice. Event E and F are defined as follows: E = Head on first toss, F = head on second toss.Find (i) $P(E \cap F)(ii)P(E \cup F)(iii)P(E/F)(iv)P(\underline{E}/F)$ Are the events E and F independent ?

35. The probability that a girl will get an admission in IIT is 0.16, the probability that she will get an admission in Government Medical College is 0.24, and the probability that she will get both is 0.11. Find the probability that (i) She will get atleast one of the two seats (ii) She will get only one of the two seats.

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QUESTION AND ANSWER
UNIT: 4

1. Define Null Hypothesis with Example.
2. What is alternate hypothesis? Give an example
3. State the meaning of degrees of freedom in hypothesis testing.
4. Explain level of significance (α) in large sample tests.
5. Define Type I error with its probability notation
6. What is Type II error? Provide its formula
7. State the Z-test formula for single population mean.
8. State Z-test statistic for difference between two sample means
9. A product analyst wants to compare the average session duration between Androidand iOS users of a mobile app.

$Platform$	n	$X(minutes)$	$s(minutes)$
$Android$	500	12.8	3.5
IOS	450	14.1	3.1

Assuming large samples, compute the Z-statistic to test whether iOS users have significantly higher session duration than Android users. (Technology Company (Product Analytics)).

10. A sample of 400 individuals is found to have a mean height of 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height of 67.39 inches and standard deviation 1.30 inches?
11. A sample of 900 members has a mean 3.5 cms and S.D. 2.61 cms. Is the sample from a large population of mean 3.25 sms and S.D 2.61 cms?
12. A random sample of 1,600 ball bearings has a mean diameter of 50.15 mm and standard deviation 2.6 mm. Can we conclude at 1% level of significance that these bearings come from a large population having mean diameter 50 mm and standard deviation 2.6 mm?
13. A sample of 400 individuals is found to have a mean height of 67.47 inches and standard deviation 1.30 inches. Compute the Z-statistic.
14. A data analyst at a fitness-tracking company wants to understand whether the average daily step count of premium users differs from the company's benchmark value of 10,000 steps. A sample of 500 premium users shows: Sample mean = 9,820 steps Sample standard deviation = 1,450 steps. Use a Z-test for sample mean to calculate the Z-statistic.(Scalar AIML / NxtWave / Google Data Analytics).
15. The mean breaking strength of cables supplied by a manufacturer is 1800 with a S.D of 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cables has increased. In order to test this, claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance?
16. An ambulance service claims that it takes on an average 8.9 minutes to reach its destination in emergency calls. To check on this, claim the agency which licenses ambulance services has them timed on 50 emergency calls, getting a mean of 9.3 minutes with a standard deviation of 1.6 minutes. What can they conclude at 5% level of significance?
17. A random sample of 625 bulbs manufactured by a company has an average life of 1,210 hours with a standard deviation of 80 hours. Can we conclude that these bulbs are drawn from a large population having mean life 1,200 hours and standard deviation 80 hours? Compute the Z-statistic and test the hypothesis at 5% level of significance.
18. In a study of the effect of chemical on the labourers in a chemical unit, the following results were obtained on their systolic blood pressures. Compute the pooled variance.

$Male$	$Exposedgroup$	$Controlledgroups$
NO	250	55
$Mean$	117.5	121.6
$S.D$	10.58	10.82

19. A random sample of 200 villages from Coimbatore district gives mean population per village at 485 with a S.D of 50. Another random sample of the same size from the same district gives the mean population per village at 510 with a S.D of 40. Is the difference between the mean values given by the two samples statistically significant? Justify your answer. Assume it is not known.

20. Explain the procedure of test of significant difference between the population mean and Sample mean for large sample test

21. A global streaming platform (similar to Netflix) historically claims that the average daily watch time per user is 92 minutes with a known population standard deviation of 28 minutes. After launching a new recommendation engine, the analytics team wants to verify if the average watch time has changed. They randomly sample 2,500 active users and record the following data:

$Parameter$	$Value$
$ClaimedPopulationMean(\mu_0)$	$92.0 minutes$
$KnownPopulationStandardDeviation(\sigma)$	$28.0 minutes$
$SampleSize(n)$	$2,500 users$
$SampleMean(\bar{x})$	$94.6\ minutes$
$LevelofSignificance(\alpha)$	$5 per$

Perform the large-sample Z-test (using the known population σ = 28 minutes) to test whether the average daily watch time has significantly changed after the new recommendation engine. (SCALAR AIML, NXT Wave 2024–2025).

22. A multinational e-commerce company claims that the average time customers spend on its mobile app is 12 minutes per session, with a population standard deviation of 3.8 minutes.To optimize server capacity, the data analytics team collects a random sample of 1,600 user sessions after a recent app update and finds: Sample mean time = 12.45 minutes Sample standard deviation = 3.9 minutes. Using the known population standard deviation of 3.8 minutes. Perform a hypothesis test at the 5% significance level to determine whether the average session time has changed after the update. (Designed for SCALAR Data Science, SCALAR AIML, NXT Wave, Google Data Analytics Certificate, IBM Data Science Professional Certificate, and Microsoft Power BI Data Analyst exams)

23. A/B test on a shopping website: Version A (control): 1,500 users, average time on site = 4.8 minutes, s = 2.1 min Version B (new design): 1,350 users, average time on site = 5.2 minutes, s = 2.3 min Test at 5% significance whether the new design increases average time spent on the site. (Use two-tailed test first, then state if one-tailed is more appropriate.). (Google Data Analytics Certificate – Course 8 (2024)).

24. A retail chain compares customer satisfaction scores (0–10) between online and in-store purchases. Online: n = 2,000, mean = 8.45, s = 1.6 In-store: n = 1,800, mean = 8.30, s = 1.7 Using 5% significance level, is there evidence that online customers are more satisfied? (Microsoft PL-300 Power BI Analyst Exam (2024)).

25. Two machine-learning models were trained on the same dataset. Accuracy on validation set: Model X (TensorFlow): 1,024 samples, mean accuracy = 92.3%, s = 3.8% Model Y (PyTorch): 900 samples, mean accuracy = 91.5%, s = 4.1% At α = 0.01, test whether Model X has significantly higher accuracy than Model Y. (IBM Data Science Professional Certificate – Final Project Style).

26. An e-commerce company runs a promotion in two regions: Region East: 2,500 orders, average order value = $78.40, σ ≈ $22 Region West: 2,200 orders, average order value = $81.20, σ ≈ $23 Test at 1% level whether the average order value differs between the regions. (AWS Certified Data Analytics – Specialty (2024)) .

27. An online advertising team is running an A/B test to compare the performance of two ad copies for a new product launch.

	$NumberofImpressions(SampleSize)$	$MeanClick-ThroughRate(CTR))$	$StandardDeviat$
$Red(Control)$	$5,000$	$4.82Per$	$2.1pe$
$Blue$	4800	$5.31per$	$2.3pe$

Using a large-sample Z-test at 5% level of significance, answer the following:
(a) State the null and alternative hypotheses to test whether the new Blue ad copy performs significantly better. (b) Compu statistic (c) State the critical value and decision rule. (d) Make the statistical decision and write the conclusion. (e) Give one recommendation to the marketing team based on the result. (Google Data Analytics (2025 Mock Exam)

28. Meta (Facebook) is testing a new feed algorithm to increase user engagement. They randomly assigned users to two groups and measured Average Daily Active Usage (DAU) in minutes.

$Group$	$SampleSize(Users)$	$MeanDAU(minutes)$	$StandardDeviation(minutes)$
$Control(OldAlgorithm)$	$3,000$	42.6	15.4
$Treatment(NewAlgorithm)$	$2,700$	45.1	16.2

Using a large-sample Z-test at 5% significance level (α = 0.05), test whether the new algorithm significantly increases daily usage. Answer the following: (a) State the null (H_0) and alternative (H_1) hypotheses. (b) Compute the Z-statistic (show complete calculations). (c) What is the critical value for this one-tailed test? (d) State your statistical decision and conclusion. (e) What recommendation would you give to the product team? (Meta (Facebook) Data Analyst Mock Test (2024)).

29. Samples of students were drawn from two universities in kgm, means and standard deviations are calculated. Make large sample test to the significance of the difference between the means.

	$Samplesize$	$Mean$	$S.D.$
$UniversityA$	400	55	10
$UniversityB$	100	57	15

30. The average hourly wage of a sample of 150 workers in a plant A was Rs. 2.56 with a S.D. of Rs. 1.08. The average wages of a sample of 200 workers in a plant B was 2.87 with a S.D of Rs. 1.28. Can an applicant safely assume that the hourly wages paid by plant B are lower than those paid by plant A? use 1% level f significance.

31. The electric light bulbs of manufacturer A, have mean life time of 1600 hours with a S.D of 200 hours, while those of manufacturer B have a mean life time of 1400h with a S.D of 100 hr. If random samples of 125 bulbs from each manufacturer are selected, then what is the probability that the bulbs will have a mean lifetime that is at least (a) 160 and (b) 250h more than B bulbs?

32. Two samples of cables of sizes each 100 are taken in order to test the breaking strength and their mean and standard deviations are given.

	$Samplesize$	$SampleMean$	$SampleSD.$
$CableI$	100	1925	1905
$CableII$	100	40	30

Test whether mean breaking strength of the two cables are equal at 1% level of significance.

33. A food-delivery app tests two recommendation algorithms: Algo 1: 1,600 deliveries, average delivery time = 32.8 minutes, s = 8.2 min Algo 2: 1,400 deliveries, average delivery time = 30.9 minutes, s = 7.9 min Using 5% significance, decide if Algo 2 significantly reduces delivery time. (SCALAR AIML / NXT Wave Certification (2024–2025).

34. A global e-commerce company claims that the average delivery time for standard shipments in Europe is 48 hours with a known population standard deviation of 12 hours. To monitor SLA (Service-Level Agreement) compliance, the Data Analytics team randomly selects 1,600 delivery records from last month.

$Parameter$	$Value$
$ClaimedPopulationMean(\mu_0)$	$48.0 hours$
$KnownPopulationStandardDeviation(\sigma)$	$12.0 hours$
$SampleSize(n)$	$1,600$
$SampleMeanDeliveryTime(\bar{x})$	$49.2 hours$
$LevelofSignificance(\alpha)–Part(a)–(d)$	0.05
$LevelofSignificance–Part(e)$	0.01

.Using the large-sample Z-test procedure and the known population σ = 12 hours, Answer the follow (a)State the null and alternative hypotheses (b) Compute the Z-statistic (c) Test the hypothesis at 5% level of significance and state the statistical conclusion. (d) If the company wants to be 99% confident (α = 0.01), would the conclusion change? Perform the calculation and justify. (IBM Data Science Capstone 2024–2025).

35.

An international ride-hailing company (similar to Uber) operates in two cities: City A and City B. The data science team wants to determine whether the average ride fare is the same in both cities after a new dynamic-pricing algorithm was rolled out. From the transaction database, they extract two independent random samples:

$City$	$SampleSize(n)$	$SampleMeanFare(USD)$	$SampleStandardDeviation(USD)$
$CityA$	$1,200 rides$	$Rs 18.40$	$Rs 5.10$
$CityB$	$980 rides$	$Rs 17.85$	$Rs 5.30$

. Assume the population standard deviations are approximately equal to the sample values (valid for large samples). Perform a two-sample large-sample Z-test at 5% significance level to decide if there is a significant difference in the average ride fare between the two cities. Compute the following. (a) State H_0 and H_1. (b) Calculate the Z-statistic (c) What is the critical value? (d) State the decision and conclusion.

Subject: SUPPORTIVE / ALLIED MATHEMATICS - 2: DISCRETE MATHEMATICS WITH PROBABILITY AND HYPOTHESIS TESTING **Code:** 25UCU305

QUESTION AND ANSWER
UNIT: 5

1. Define t test and give the formula for t test statistic for single mean.

2. State any four properties of student's 't' distribution.

3. A sample of 10 students has an average test score of 72, and the sample standard deviation is 8. Test whether the mean score is different from the hypothesized population mean of 75. Compute the t-test statistic.

4. A food-delivery company claims that its average delivery time in a metropolitan city is 30 minutes or less. To verify this claim, a quality-control team randomly selected 16 orders and recorded the following delivery times (in minutes): 32, 28, 35, 39, 27, 31, 33, 29, 36, 30, 34, 31, 38, 26, 33, 30. At α = 0.05, perform a one-sample t-test to determine whether the true mean delivery time is significantly greater than 30 minutes. Critical t-value from t-table: $t_{0.05,\ 15} = 1.753$.

5. State t-test formula for difference between two means

6. What is chi-square test? State its uses.

7. Give the formula for goodness-of-fit in chi-square test

8. Give the formula for independence of attributes in chi-square test

9. State any four properties of χ^2 distribution

10. Explain F-test and its degrees of freedom

11. State the formula for finding degrees of freedom (Welch–Satterthwaite approximation).

12. A data science team is comparing the effectiveness of two machine learning models on a small validation dataset.

$Parameter$	$ModelA(Baseline)$	$ModelB(New)$
$SampleSize(n)$	24	19
$MeanAccuracy$	78.5	82.3
$SampleStandardDeviation$	4.8	6.2

Compute the degrees of freedom using the Welch–Satterthwaite approximation. (Typical Scaler Data Science / AI-ML Certification Question).

13. Two laboratories measure the concentration of a chemical in the same solution. Samples taken from each lab give the following results: Two laboratories measure the concentration of a chemical in the same solution. Samples taken from each lab give the following results:

Laboratory	**Sample Size** (n)	**Sample Variance** (s^2)
Lab X	15	36
Lab Y	13	16

Test at the 0.05 significance levelwhether the two population variances are equal. Critical value F(0.025,14,12)=3.206.Test at the 0.05 significance levelwhether the two population variances are equal. Critical value F(0.025,14,12)=3.206.

14. A quality engineer wants to determine if two production lines (Line P and Line Q) are operating with the same process variability in terms of product weight (in grams). Independent random samples were taken with the following results: Calculate the F-statistic.

15. Give the procedure for finding significant difference between population mean and sample mean for t test.

16. Give the procedure for finding significant difference between two sample means for t test.

17. A manufacturer claims that the average lifetime of a LED bulb is5000 hours. A sample of10 bulbsgives an average lifetime of4800 hours, with a sample standard deviation of 300 hours. Using a one-sample t-test, compute thet-test statistic.

18. In a data science project analyzing employee performance, a sample of 16 analysts from a firm yields an average daily output of 42 reports, with a standard deviation of 3.8 reports. The industry standard mean output is 40 reports per day. Conduct a one-sample t-test at the 1% significance level to assess if the firm's analysts exceed the standard.Employee Productivity Analysis (Representative of Scaler Data Science Course Exercises).

19. A marketing team tests a new ad campaign by surveying 20 customers, finding an average satisfaction score of 7.8 out of 10, with a standard deviation of 1.2. The historical population mean satisfaction score is 7.0. Using a 5% significance level, perform a one-sample t-test to determine if the campaign has improved satisfaction. Marketing Campaign Effectiveness (Representative of NxtWave Data Analytics Course Exercises).

20. A pharmaceutical company is evaluating a new formulation of a drug intended to lower systolic blood pressure more effectively than the current placebo-controlled baseline.

$Parameter$	$PlaceboGroup$	$NewDrugGroup$
$SampleSize(n)$	15	19
$MeanreductioninsystolicBP(mmHg)$	2.80	7.40
$SampleStandardDeviation(mmHg)$	3.10	4.20

Conduct a two-tailed Welch's t-test at α = 0.05 to determine whether the new drug produces a statistically significant difference in mean blood-pressure reduction compared to the placebo Assume unequal population variances.

21. An e-commerce platform conducted an A/B test to evaluate a new recommendation engine expected to increase user engagement.

$Parameter$	$ControlGroup(OldAlgorithm)$	$TreatmentGroup(NewAlgorithm)$
$SampleSize(n)$	22	18
$Meansessionduration(minutes)$	6.20	8.10
$SampleStandardDeviation(minutes)$	1.80	2.30

Using Welch's two-sample t-test at α = 0.05, test the hypothesis that the new recommendation algorithm leads to a statistically significant increase in average session duration. Assume population variances are unequal.(tcrit≈1.694). (Scaler Data Science, Scaler AI/ML, NxtWave CCBP

22. A retail chain manager claims that customer footfall in their store is evenly distributed across the six days it is open (Mond Saturday), i.e., approximately 16.67% of weekly customers visit on each day. Over a period of 8 weeks, a total of 720 custom were recorded with the following observed distribution:

Day of the Week	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**T**
Observed Frequency (O_i)	98	102	108	110	142	160	7
Expected Frequency (E_i)	?	?	?	?	?	?	7

Using a Chi-Square Goodness-of-Fit test at α = 0.05, test whether the customer footfall is uniformly distributed across the si days. (Critical value $\chi^2_{0.05,\,5} = 11.070$) (Scaler Data Science,Scaler AI/ML).

23. A retail bank wants to test whether \Gender (Male / Female) is independent of Preference for Digital Banking Channel (Mobile App / Net Banking / Branch Visit).A random sample of 500 customers produced the following contingency table:

$Gender\ Channel$	$MobileApp$	$NetBanking$	$BranchVisit$	$RowTotal$
$Male$	120	85	45	250
$Female$	140	60	50	250
$ColumnTotal$	260	145	95	500

Perform a Chi-Square Test of Independence at α = 0.05 to determine if Gender and Preferred Banking Channel are independent. critical value = 5.991. (Very commonly repeated in Scaler DS & AIML 2023–2025 batches).

24. An ed-tech company wants to check whether Student

$EmploymentStatus\ CourseOutcome$	$Completed$	$DroppedOut$	$RowTotal$
$Full-timeEmployed$	110	40	150
$Part-timeEmployed$	95	55	150
$Unemployed$	120	30	150
$Student$	105	45	150
$ColumnTotal$	430	170	600

Using α = 0.05, Conduct a Chi-Square Test of Independence. (df = 3, critical value = 7.815). (Standard in NxtWave CCBP 4.0 & Scaler final exams).

25. A machine learning team is evaluating two versions of a sentiment-analysis model on separate small test sets. The following summary statistics were obtained: Parameter

$Parameter$	$ModelX(current)$	$ModelY(Proposed)$
$SampleSize(n)$	26	21
$MeanF1-Score(minutes)$	84.6	88.9
$SampleStandardDeviation(Percentage)$	5.4	7.1

. Assuming unequal population variances, compute the approximate degrees of freedom for Welch's two-sample t-test using the Welch–Satterthwaite approximation. (Scaler Data Science).

26. A retail chain wants to test whether the proportion of customers who purchase "Product X" is the same across its three geographical regions (North, East, and West). Random samples of customers were taken from each region with the following results:

Region	**Purchased Product X (Yes)**	**Did Not Purchase (No)**	**Row Total**
North	85	115	200
East	72	128	200
West	96	104	200
Column Total	**253**	**347**	**600**

Using a Chi-Square test at α = 0.05, determine whether the proportion of customers purchasing Product X is homogeneous (the same) across the three regions. (df = 2, critical value = 5.991).

27. An ed-tech platform wants to check whether the course completion rate is the same across four different marketing channels (Google Ads, Facebook Ads, Email Campaign, and Referral)

$MarketingChannel\ Channel$	$CompletedCourse$	$DroppedOut$	$RowTotal$
$GoogleAds$	120	80	200
$FacebookAds$	105	95	200
$EmailCampaign$	135	65	200
$Referral$	92	108	200
$ColumnTotal$	452	348	800

Perform a Chi-Square test forhomogeneity at α = 0.05 to test if the course completion rate is identical across the four marketing channels. (df = 3, critical value = 7.815). (Standard in Almost Every Certification Final Exam)

28. A company wants to check whether two machines produce items with thesame variability. Two samples are taken: Machine A:Sample size n_1= 10 Sample variance s_1^2=25 Machine B: Sample size n_2= 12Sample variance s_2^2=10 Test whether the population variances are equal at the 5 percentage level of significance. Critical value F_(0.025,9,11)=4.03.F_(0.025,14,12)=3.206.

29. A data science team is analyzing user engagement for a mobile app. The product manager claims that the average daily time spent by users is at least 35 minutes. To check this claim, a data analyst collects a random sample of 12 users, and finds the following time spent per day (in minutes): Sample data: 34, 30, 28, 36, 40, 32, 38, 29, 31, 33, 37, 35. Using a one-sample t-test, test whether the mean daily usage is less than 35 minutes at the 5% significance level. State H_0andH_1. 2. Compute the sample mean and sample standard deviation. Calculate the t-test statistic. State whether the claim is supported or rejected. Critical t-value from t-table: $t_{0.05,\ 11} = 1.796$. (similar to SCALAR AIML, SCALAR Data Science, NxtWave, etc.).

30. An e-commerce company wants to test whether a new website design (Version B) leads to a statistically significant increase in average time spent on the site compared to the old design (Version A).

$Parameter$	$VersionA(OldDesign)$	$VersionB(NewDesign)$
$SampleSize(n)$	30	35
$SampleMean(\bar{x})$	$4.8minutes$	$5.7minutes$
$SampleStandardDeviation(s)$	$1.4minutes$	$1.9minutes$
$SampleVariance(s^2)$	1.96	3.61

Perform a two-sample t-test at α = 0.05 to determine whether the new design significantly increases the average time spent. Assume the population variances are unequal. (tcrit≈1.6699)(Frequently used in Scaler DS & AIML batches).

31. An online retail company introduced a redesigned checkout interface to reduce transaction abandonment by shortening completion time

$Parameter$	$OldCheckoutLayout(A)$	$NewCheckoutLayout(B)$
$SampleSize(n)$	25	21
$Meancheckouttime(seconds)$	78.40	69.20
$SampleStandardDeviation(seconds)$	12.50	15.80

At a 5% significance level, use Welch's two-sample t-test (unequal variances assumed) to test whether the new layout significantly reduces the average checkout time (one-tailed test). Provide the full solution including the t-statistic, approximate degrees of freedom, decision rule, and practical interpretation.(tcrit≈t0.95,df≈37.84≈1.68613) (Scaler Data Science, Scaler AI/ML, and NxtWave certificat

32. A data science team at an e-commerce company believes that customer complaints received through their support portal follow a uniform distribution across the five working days (Monday to Friday), i.e., 20% of complaints each day.They recorded the complaints over 10 weeks (total 300 complaints) and obtained the following observed frequencies: Day Monday Tuesday Wednesday Thursday Friday Total Observed (O_i) 72 54 48 51 75 300 Expected (E_i) ? ? ? ? ? 300 At α = 0.05, perform a Chi-Square Goodness-of-Fit test to determine whether the complaints are uniformly distributed across the five weekdays. (From Chi-Square table: $\chi^2_{0.05,4} = 9.488$). (Scaler Data Science, Scaler AI/ML, NxtWave CCBP 4.0, and other certification final exams and mock tests).

33. An e-commerce company wants to test whether Customer Segment (New vs Returning) is independent of Preferred Payment Method (Credit Card, Debit Card, UPI, Cash on Delivery). A random sample of 400 recent transactions yielded the following contingency table:

$CustomerSegment\ PaymentMethod$	$CreditCard$	$DebitCard$	UPI	$CashonDelivery$	$RowTotal$
$NewCustomer$	48	62	95	45	250
$ReturningCustomer$	72	38	25	15	150
$ColumnTotal(seconds)$	120	100	120	60	400

At α = 0.05, perform a Chi-Square Test of Independence to determine whether Customer Segment and Preferred Payment Method are independent. (Critical value $\chi^2_{0.05,3} = 7.815$). (Regularly asked in Scaler Data Science, Scaler AI/ML, NxtWave CCBP 4.0/Intensive, and most international certification final assessments). Representative Proctored-Exam Problem.

34. A manufacturing company is comparing the consistency of two production machines (Machine A and Machine B) by measuring the diameter (in mm) of 10 randomly selected parts from each machine. The company wants to test whether the variances in diameters are significantly different at α=0.05. Machine A diameters:

Machine A diameters	50.1	49.8	50.3	49.9	50.2	50.0	50.4	49.7	50.1	50.0
Machine B diameters	50.5	49.5	50.7	49.6	50.8	49.4	50.6	49.3	50.9	49.2

Upper critical value: (Upper critical value: F_(0.975,9,9)≈ 4.02599.

35.

A manufacturing engineer wishes to determine whether two CNC machines (Machine X and Machine Y) produce parts with the same precision (variability in diameter). Random samples of parts are measured with the following results:

Machine	**Sample Size** (n)	**Sample Variance** (s^2)
Machine X	15	0.028
Machine Y	21	0.073

(a) State the null and alternative hypotheses. (b) Compute the F-statistic (show the formula and calculation). (c) State the critical value(s) from the F-table. (d) State the decision and conclusion. (Upper critical value: $F_{0.025,\,20,\,14} \approx 2.57$).