Dr.SNS RAJALAKSHMI COLLEGE OF ARTS AND SCIENCE
(Autonomous)
Accredited by NAAC – UGC with 'A+ Grade (Cycle IV)
(Recognised by UGC, Approved by AICTE & Affiliated to Bharathiar University)
Coimbatore- 49

DEPARTMENT OF MATHEMATICS

25UCU305: DISCRETE MATHEMATICS WITH PROBABILITY AND HYPOTHESIS TESTING

INVERSE FUNCTIONS

Dr.K.Sasikala
Assistant Professor,
Department of Mathematics

Inverse Functions

Introduction to Inverse Functions:

A function has an inverse if it is one-to-one and onto (bijective).

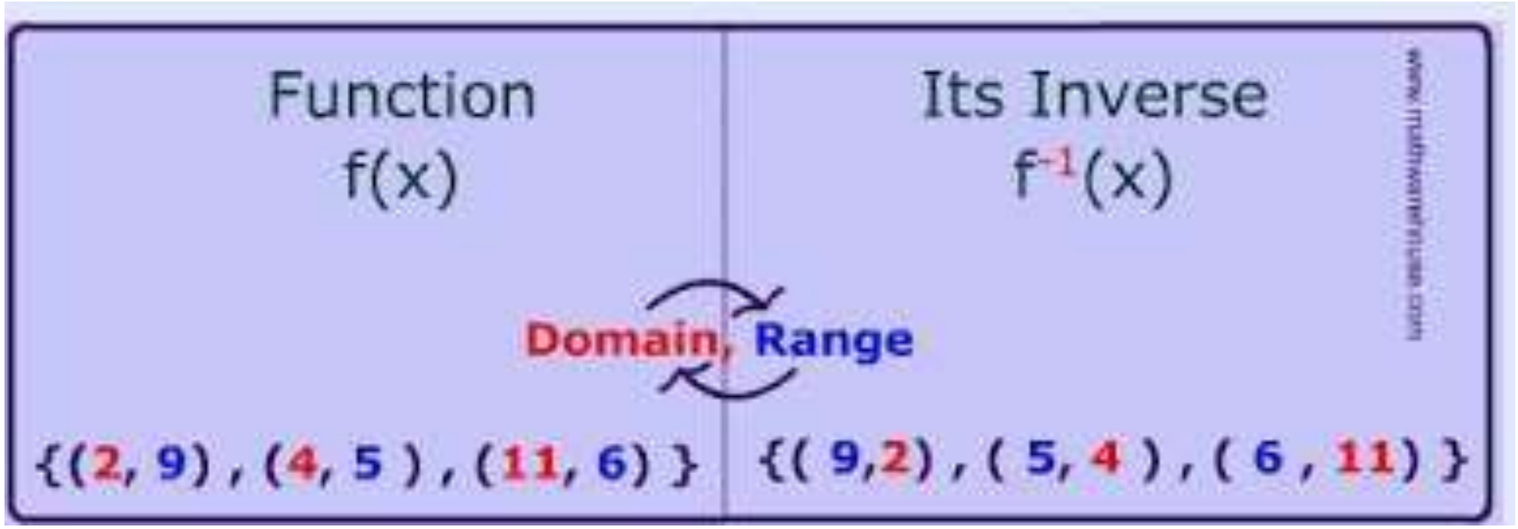
The inverse function reverses the mapping of the original function.

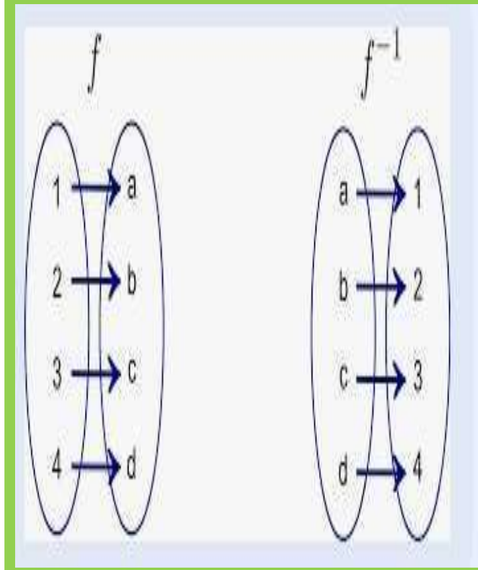
Inverse Functions

Definition:

If f: A → B is bijective, then the inverse function f^{-1}: B → A is defined as:

$$f(a) = b \quad \Rightarrow \quad f^{-1}(b) = a$$

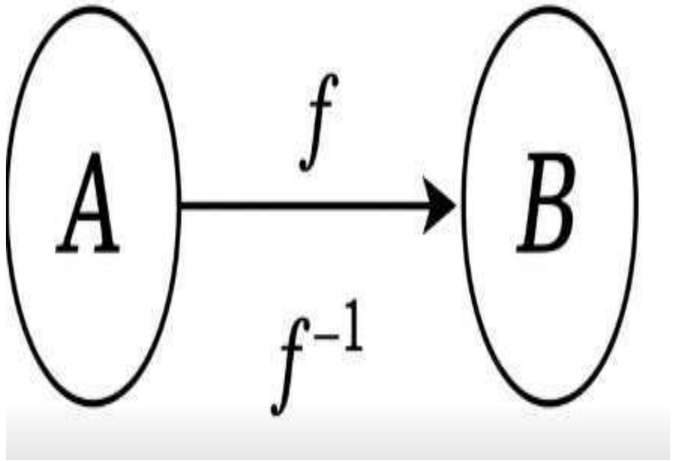


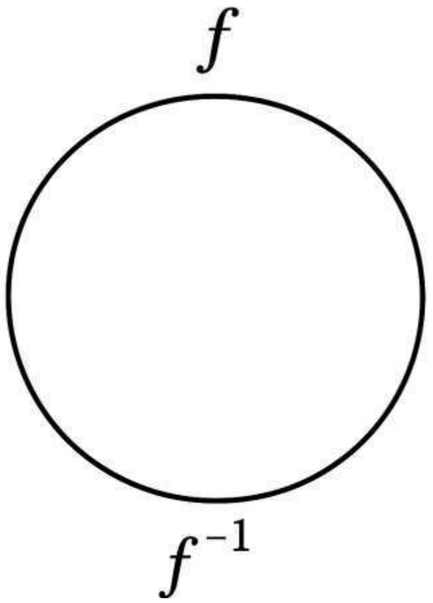


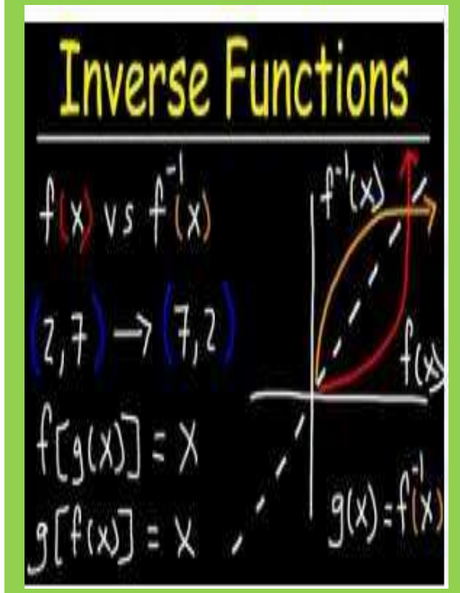
Fundamental Property

Every inverse function satisfies:

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x$$







Example 1

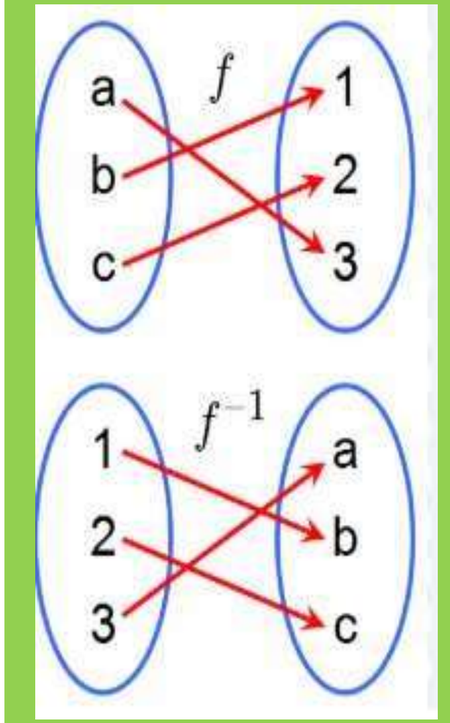
Let

f(x)=2x+3f(x) = 2x + 3f(x)=2x+3 To find its inverse:

1. **Replace f(x) with y**
2. **Swap x and y**
3. **Solve for y**

$$y = 2x + 3 \Rightarrow x = 2y + 3 \Rightarrow y = \frac{x - 3}{2}$$

$$f^{-1}(x) = \frac{x - 3}{2}$$

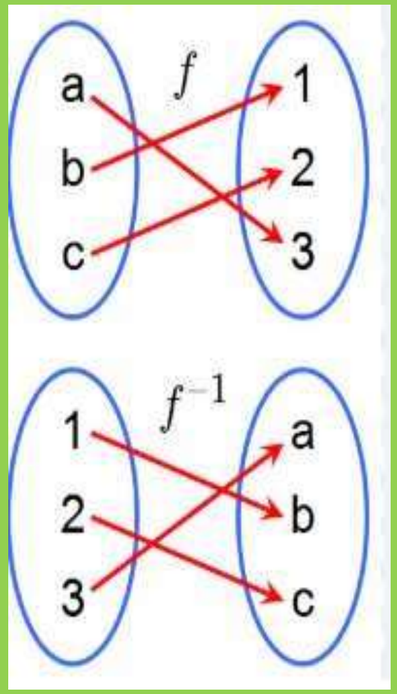


Example 2 – Table of Values

x	f(x) = x + 1
1	2
2	3
3	4

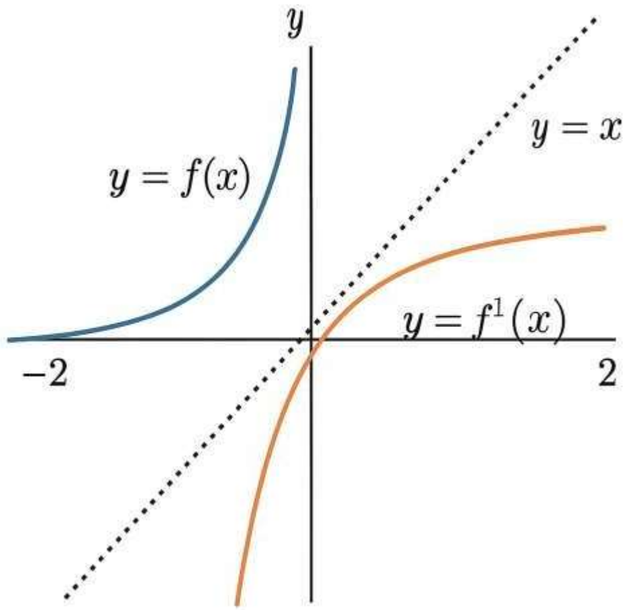
Inverse table (swap x and f(x)):

y	$f^{-1}(y)$
2	1
3	2
4	3



Graphical Interpretation:

- A function and its inverse are mirror images across the line y = x.
- Reflection property helps identify inverse visually.

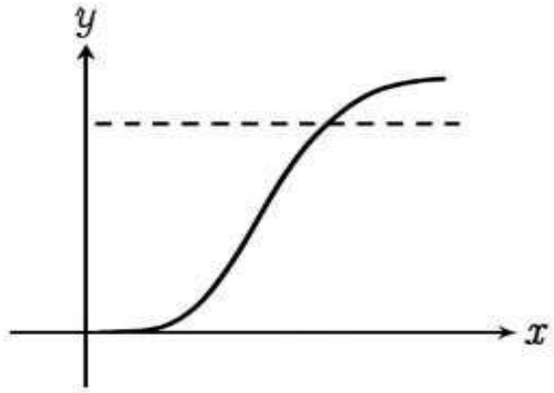


Horizontal Line Test:

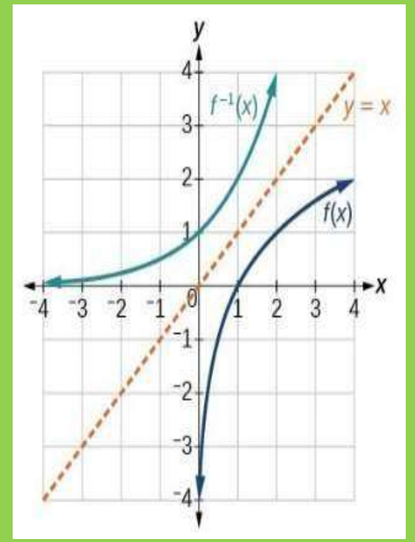
A function is invertible if and only if it is one-one.

The Horizontal Line Test:

If any horizontal line cuts the graph at most once, the function has an inverse.

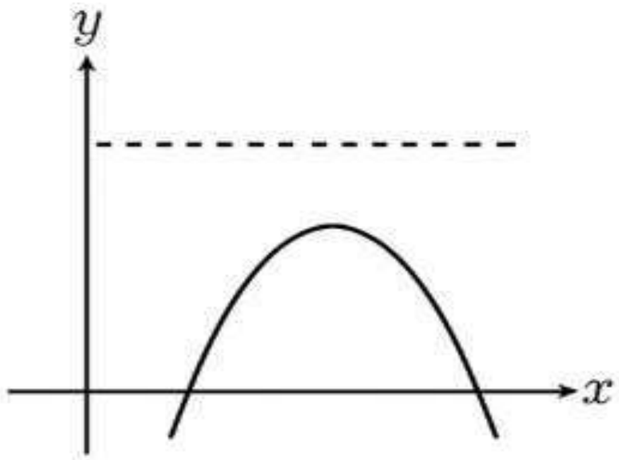


Horizontal Line Test

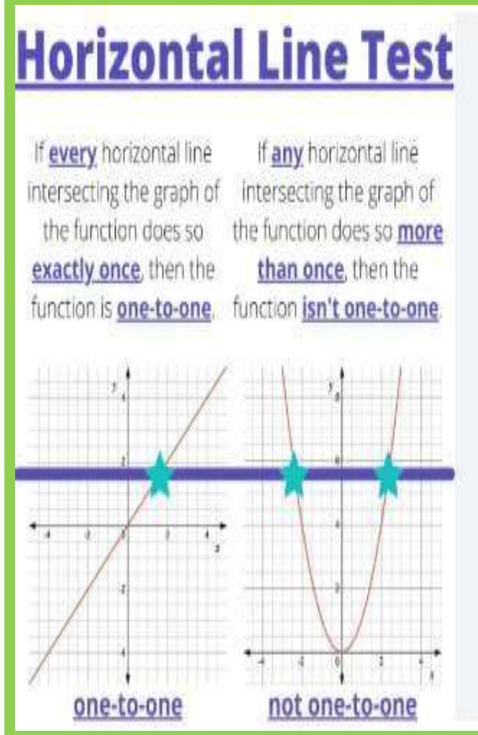


Non-Invertible Function Example

Let $f(x) = x^2$ (for all real numbers).
It is not one-one, so inverse does not exist for all reals.
But restricting the domain to $x \geq 0$ makes it invertible.



Failing Horizontal LineTest

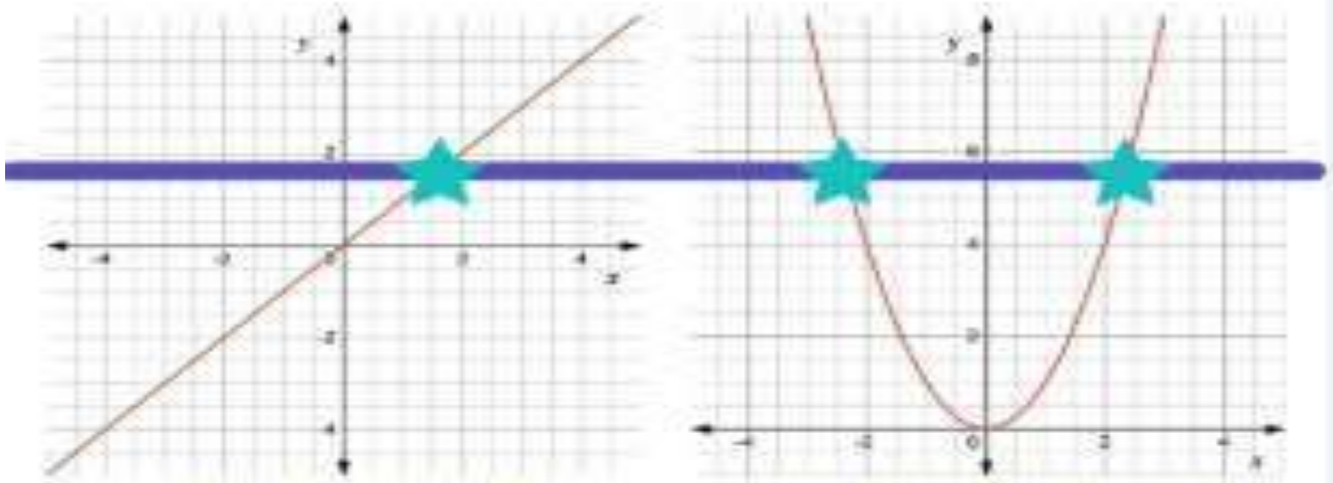


Inverse Functions

Horizontal Line Test

If **every** horizontal line intersecting the graph of the function does so **exactly once**, then the function is **one-to-one**.

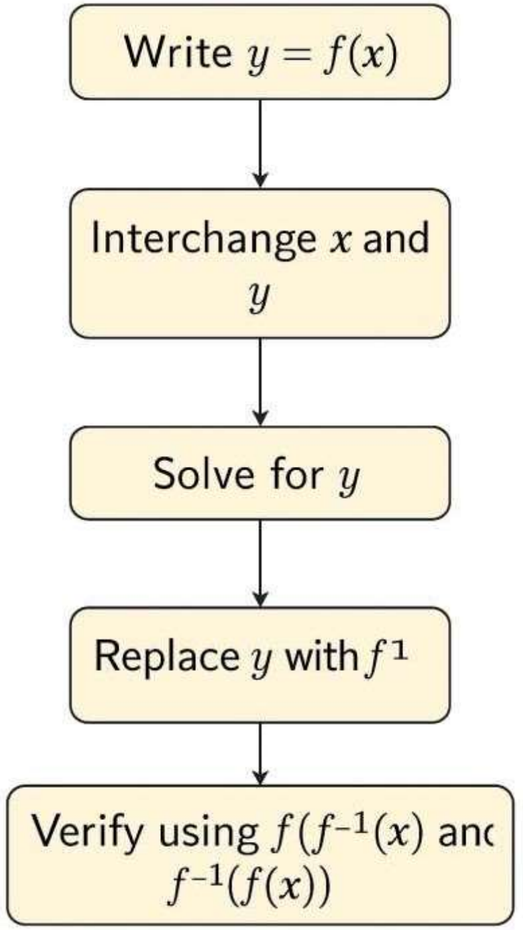
If **any** horizontal line intersecting the graph of the function does so **more than once**, then the function **isn't one-to-one**.



one-to-one **not one-to-one**

Steps to Find Inverse Function

1. Write y = f(x)
2. Interchange x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$
5. Verify using $f(f^{-1}(x))$ and $f^{-1}(f(x))$



Summary

- Inverse function reverses original mapping.
- Exists only for bijective functions.
- Represented as $f^{-1}(x)$.
- Graphically, it is reflection over $y = x$.
- Horizontal line test determines invertibility.

Thank you