Definition

R on A is an equivalence relation if it is:

- **Reflexive**
- **Symmetric**
- **Transitive**

Reflexive Property

(a, a) ∈R for all a ∈A

Example: (1,1), (2,2), (3,3)

Symmetric Property

$$(a, b) \in R \Rightarrow (b, a) \in R$$

Example: $(2,5) \Rightarrow (5,2)$

Transitive Property

(a,b) and $(b,c) \in R \Rightarrow (a,c) \in R$

Example: (1,2) and (2,3) ⇒ (1,3)

A = {1, 2, 3, 4}
Define R = “a and b have the same parity (odd/even).”
Then

Odd class: {1, 3}
Even class: {2, 4}

R is an equivalence relation because:

Reflexive: each number has same parity with itself

Symmetric: if a same-parity with b → b same-parity with a

Transitive: if a same as b and b same as c → a same as c

Example: Congruence mod n

Relation: “$a \equiv b \pmod{n}$”

Meaning:

$a-b$ is divisible by n. This is a classic equivalence relation.

Example: mod 3

$1 \equiv 4$

$2 \equiv 5$

$0 \equiv 3, 6$

Equivalence Classes

For an equivalence relation R on A,

the equivalence class of a is:

$$[a]=\{x \in A \mid (a,x) \in R\}$$

Example classes mod 3

$[0] = \{0,3,6,9\}$
$[1] = \{1,4,7\}$
$[2] = \{2,5,8\}$

Partition of a Set

- An equivalence relation on A divides A into disjoint subsets.
- These subsets are called equivalence classes.

Important facts:

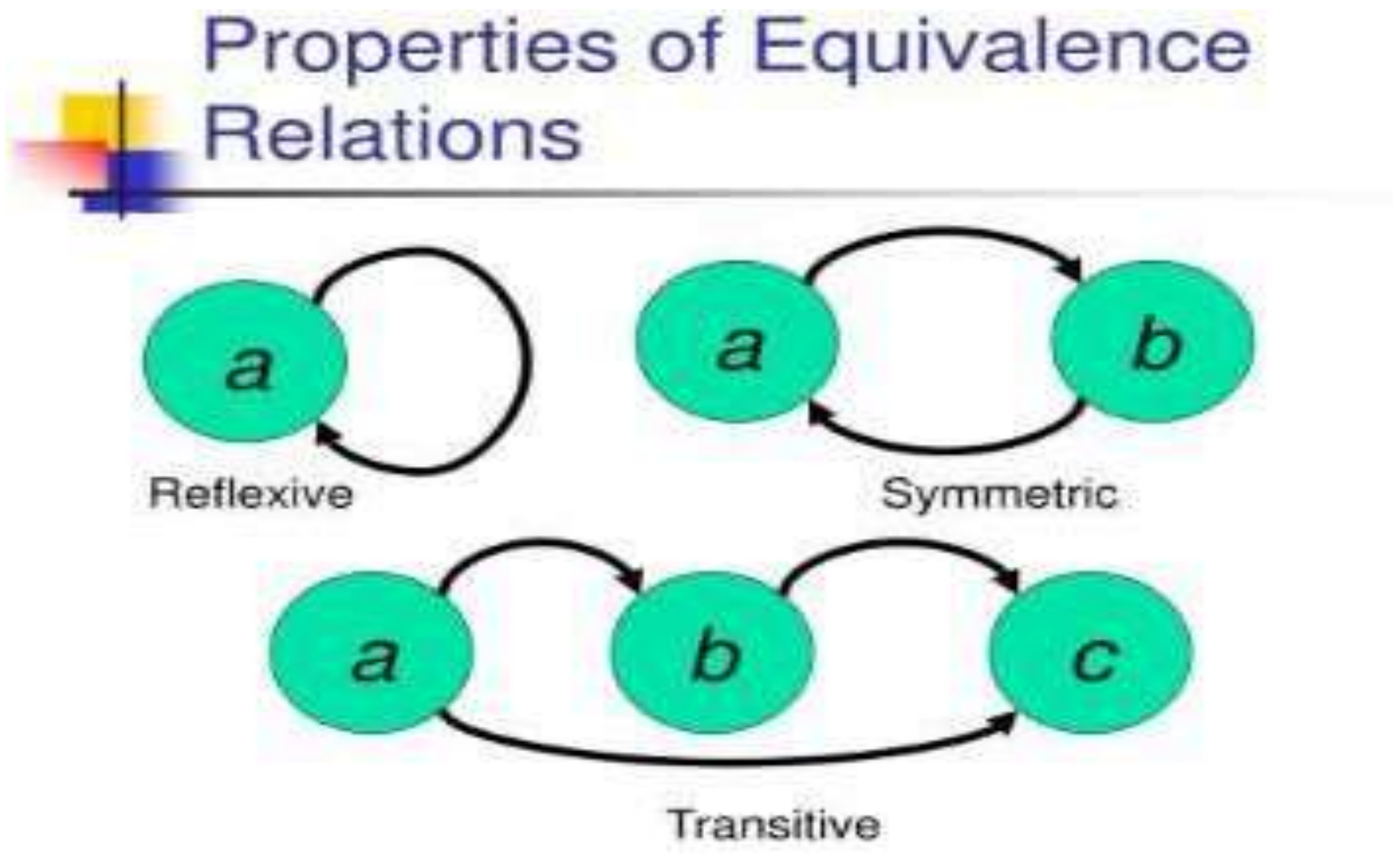
- Every element belongs to exactly one class
- Classes do not overlap
- All classes together form a partition of A

Real-Life Applications

- Grouping students by department
- Categorizing files with same extension
- Classifying shapes with same number of sides
- Congruence of triangles in geometry
- Modular arithmetic in computer science

Summary

Equivalence = Reflexive + Symmetric + Transitive



Thank You