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**Coimbatore- 49**

## DEPARTMENT OF MATHEMATICS

**25UCU305: DISCRETE MATHEMATICS WITH PROBABILITY AND HYPOTHESIS TESTING**

# MATRIX RELATIONS

**Dr.K.Sasikala**
**Assistant Professor,**
**Department of Mathematics**

# Introduction to Matrix Relations

- **A relation connects elements of one set with elements of another set.**
- **Relations are often represented using sets, diagrams, or matrices.**
- **Matrix representation is useful for computing compositions, inverses, and properties of relations.**

# Introduction to Matrix Relations

## Definition of Matrix Representation

**A relation R from set A = {$a_1$, $a_2$, …, $a_m$} to B = {$b_1$, $b_2$, …, $b_n$}**
**can be represented by an m × n matrix M(R).**

$$M_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{otherwise} \end{cases}$$

# Example

Let

A = {1, 2, 3}

B = {1, 2, 3}

Relation R = {(1,2), (2,3), (3,1)}

Matrix:

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 |
| 3 | 1 | 0 | 0 |

# Introduction to Matrix Relations

## Types of Relations (Using Matrices)

**A relation on a set can be tested using properties of its matrix:**

- **Reflexive**
- **Symmetric**
- **Antisymmetric**
- **Transitive**

# Reflexive Relation

A relation R on A is reflexive if each element is related to itself.

**Matrix condition:**
**All diagonal entries are 1.**

**Example:**

If $M_{11} = M_{22} = M_{33} = 1 \rightarrow$ Reflexive.

## Symmetric Relation

**R is symmetric if (a, b) ∈ R → (b, a) ∈ R.**

**Matrix condition:**

**Matrix must be equal to its transpose ($M = M^T$).**

# Antisymmetric Relation

**R is antisymmetric if:**

**$(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$**

**Matrix condition:**

**For $i \neq j$, if $M_{ij} = 1$ then $M_{ji}$ must be 0.**

# Transitive Relation

**R is transitive if:**

**$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$**

**Matrix test:**

**Compute Boolean product $M \odot M$**
**If $M \odot M \leq M \rightarrow$ Transitive**

# Composition of Relations

**For relations R (A → B) and S (B → C):**

**Composition R ∘ S is represented by Boolean matrix multiplication.**

$$(M(R) \odot M(S))_{ij} = \bigvee_k (M(R)_{ik} \wedge M(S)_{kj})$$

# Inverse of a Relation

**Inverse $R^{-1} = \{(b, a) : (a, b) \in R\}$**

**Matrix representation:**

**Transpose of matrix M(R).**

# Summary

**Relation matrices are binary matrices representing ordered pairs**

**Diagonal → reflexive**

**Symmetry → $M = M^T$**

**Antisymmetry → off-diagonal conditions**

**Transitivity → Boolean matrix multiplication**

**Inverse → transpose**

# THANK YOU