25UCU305: DISCRETE MATHEMATICS WITH PROBABILITY AND HYPOTHESIS TESTING

Year: I (B.Sc., CS, IT, AI-DS, DS &V, BCA, DA, Cyber Security, DevOps, FSWD)

Semester: II

Unit III: Probability

4 Marks

1. If two coins are tossed simultaneously, then find the probability of getting (i) One head and one tail (ii) at least one head.
2. When a pair of balanced dice is rolled, what are the probabilities of getting the sum (i) 7 (ii) 7 or 11 (iii) 11 or 12
3. Five mangoes and 4 apples are in a box. If two fruits are chosen at random, find the probability that (i) one is mango and the other is an apple (ii) both are of the same variety.
4. What is the chance that (i) non-leap year (ii) leap year should have fifty-three Sundays?
5. If A and B are mutually exclusive events $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{8}$ then find $(i)\ P(\underline{A})(ii)P(A \cup B)(iii)P(\underline{A} \cup \underline{B})$
6. If A and B are two independent events such that $P(A \cup B) = 0.6, P(A) = 0.2. find\ P(B)$.
7. If $P(A) = 0.5, P(B) = 0.8\ and\ P\left(\frac{B}{A}\right) = 0.8$ find $P\left(\frac{A}{B}\right)$.

Given that P(A) = 0.35, P(B) =0.73, and $P(A \cap B) = 0.14$. find

$$(i)P(A \cup B), (ii)P(\underline{A} \cap B)\ (iii)P(A \cap \underline{B})$$

8. A die is rolled. If it shown an odd number, then find the probability of getting 5.
9. A number is selected from the set $\{1, 2, 3, \ldots 20\}$. The probability that the selected number is divisible by 3 or 4.
10. Two units are chosen from a lot containing twelve units, of which four are defective. Find the probability that at least one of the units is defective.
11. If A and B are two independent events such that$P(A) = 0.5\ and\ P(A \cup B) = 0.8, Find\ P(B)$
12. An urn contains 10 white and 5 black balls. While another urn contains 3 white and 7 black balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are white.
13. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is chosen is a prime or multiple of 8?

6 Marks

14. A Cricket club has 16 members of whom only 5 can bowl. What is the probability that in a team of 11 members at least 3 bowlers are selected?

15. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it willget both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards?
16. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane in the first, second, third, and fourth shot are respectively 0.2, 0.4, 0.2 and 0.1. Find the probability that the gun hits the plane.
17. X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?
18. The probability that a car being filled with petrol will also need an oil change is 0.30; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.15.
    (i) If the oil had to be changed, what is the probability that a new oil filter is needed?
    (ii) If a new oil filter is needed, what is the probability that the oil has to be changed?
19. There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?
20. A firm manufactures PVC pipes in three plants viz., X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production,
    (i) find the probability that the selected pipe is a defective one.
    (ii) If the selected pipe is a defective, then what is the probability that it was produced by plant Y?
21. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television.
22. A factory has two machines I and II. Machine I and II produce 30% and 70% of items respectively. Further 3% of items produced by Machine I are defective and 4% of items produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.
23. A consulting firm rents car from three agencies such that 20% from agency X, 30% from agency Y and 50% from agency Z. If 90% of the cars from X, 80% of cars from Y and 95% of the cars from Z are in good conditions

(i) What is the probability that the firm will get a car in good condition? Also

(ii) If a car is in good condition, what is probability that it has come from agency Y?

24. Two thirds of students in a class are boys and rest girls. It is known that the probability of a girl getting a first class is 0.75 and that of a boy is 0.70. Find the probability that a student chosen at random will get first class marks.
25. A husband and wife appear in an interview for two vacancies in the same post. The probability of husbands' selection is 1/6 and that of wife's selection is 1/5. What is the probability that (i) both of them will be selected (ii) only one of them will be selected (iii) none of them will be selected?
26. The probability that a new ship will get an award for its design is 0.25, the probability that it will get an award for the efficient use of materials is 0.35, and that it will get both awards is 0.15. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards?
27. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then find the probability of neither A nor B occurring.

10 Marks (Case Study)

28. The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25, and the probability that she will get both is 0.07. Find the probability that (i) she will get at least one of the two jobs (ii) she will get only one of the two jobs.
29. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{4} and \frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?
30. A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?
31. The chances of *X, Y* and *Z* becoming managers of a certain company are 4: 2: 3. The probabilities that bonus scheme will be introduced if *X, Y* and *Z* become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that *Z* was appointed as the manager?
32. A consulting firm rents car from three agencies such that 50% from agency *L*, 30% from agency *M* and 20% from agency *N*. If 90% of the cars from *L*, 70% of cars from *M* and 60% of the cars from *N* are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency *N*?

33. A coin is tossed twice. Event E and F are defined as follows: E = Head on first toss, F = head on second toss. Find $(i)P(E \cap F)(ii)P(E \cup F)(iii)P\left(\frac{E}{F}\right)(iv)P\left(\frac{E}{\overline{F}}\right)$ (v) Are the events E and F independent ?
34. The probability that a girl will get an admission in IIT is 0.16, the probability that she will get an admission in Government Medical College is 0.24, and the probability that she will get both is 0.11. Find the probability that (i) She will get atleast one of the two seats (ii) She will get only one of the two seats.