

25UCU305: DISCRETE MATHEMATICS WITH PROBABILITY AND HYPOTHESIS TESTING

Year: I (B.Sc., CS, IT, AI-DS, DS &V, BCA, DA, Cyber Security, DevOps, FSWD)

Semester: II

UNIT:I - RELATIONS AND FUNCTIONS

4 Marks

1. Define Binary relations and Reflexive Relations
2. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be a function defined on the x set of real numbers. Find $f \circ g$ and $g \circ f$ where $f(x) = x^2 - 2$ and $g(x) = x + 4$.
3. Define Equivalence relations and give example for it.
4. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be a function defined on the x set of real numbers. Find $f \circ g$ and $g \circ f$ where $f(x) = 4x^2 - 1$ and $g(x) = x + 1$.
5. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be a function defined on the x set of real numbers. Find $f \circ g$ and $g \circ f$ where $f(x) = x - 6$ and $g(x) = x^3$.
6. Let $f(x) = x^2 - 5x + 6$. Find $f(1), f(2), f(0)$.
7. Express the relation R from $A = \{3,5,7,9,11\}$ into $B = \{2,6,8,10\}$ as a set of ordered pairs defined by the sentence "R: x is one less than y"
8. Find the Cartesian Product of $A \times B, B \times A$ for the sets $A = \{1,3,4\}$ and $B = \{5,6\}$.
9. Check whether the following functions are one-to-one and onto. $f: N \rightarrow N$ defined by $f(n) = n + 2$. (Draw an arrow diagram)
10. Find the value of k such that $f \circ g = g \circ f$ if $f(x) = 3x + 2, g(x) = 6x - k$.
11. Find the value of k such that $f \circ g = g \circ f$ if $f(x) = 2x - k, g(x) = 4x + 5$.
12. Let $f(x) = x^2 - 1$ Find (i) $f \circ f$ (ii) $f \circ f \circ f$
13. A function f is defined by $f(x) = 3 - 2x$. find x such that $f(x^2) = (f(x))^2$.
14. If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \rightarrow B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

6 Marks

15. Let $S = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 2), (1, 3), (3, 1)\}$.
 - (i) Is R reflexive? If not, state the reason and write the minimum set of ordered pairs to be included to R so as to make it reflexive.
 - (ii) Is R symmetric? If not, state the reason, write minimum number of ordered pairs to be included to R so as to make it symmetric and write minimum number of ordered pairs to be deleted from R so as to make it symmetric.
16. If $f(x) = 2x + 3, g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.
17. Define (i) Binary Relation (ii) Inverse Relation (iii) Equivalence relation
18. Let $A = \{2, 5, 6\}$ and let r be the relation $\{(2, 2), (2, 5), (5, 6), (6, 6)\}$ on A, Find the adjacency matrix r and r^2 .
19. If $A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$ then show that $A \times A = (B \times B) \cap (C \times C)$.

20. Let $A = \{a, b, c\}$. What is the equivalence relation of smallest cardinality on A? What is the equivalence relation of largest cardinality on A?
21. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$
Are the following true
- f is a relation from A to B
 - f is a function from A to B? Justify your answer.
22. Let f be a function defined by $f(x) = 3x + 2$ then (i) Find the images of 1, 2, 3 (ii) find the pre-images of 29, 53.
23. Let the function f and g on the real numbers be defined by $f(x) = x^2 + 2x - 3$, $g(x) = 3x - 4$. Find $f \circ g$ and $g \circ f$.
24. In the set Z of integers, define mRn if $m - n$ is divisible by 7. Prove that R is an equivalence relation.
25. The function for exchanging American dollars for Singapore Dollar on a given day is $f(x) = 1.3x$ where x represents the number of American dollars. On the same day the function for exchanging Singapore Dollar to Indian Rupee is $g(y) = 69.7y$, where y represents the number of Singapore dollars. Write a function which will give the exchange rate of American dollars in terms of Indian rupee.
26. The owner of a small restaurant can prepare a particular meal at a cost of Rupees 100. He estimates that if the menu price of the meal is x rupees, then the number of customers who will order that meal at that price in an evening is given by the function $D(x) = 250 - x$. Express his day revenue, total cost and profit on this meal as functions of x .
27. A simple cipher takes a number and codes it, using the function $f(x) = 4x + 5$. Find the inverse of this function, determine whether the inverse is also a function.
28. Let f be a function from R to R defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f

10 Marks (Case Study)

29. Explain the types of relations with examples.
30. Let $X = \{1, 2, 3\}$ and f, g, h and s be function from X to X given by
 $f: \{(1, 2), (2, 3), (3, 1)\}$, $g: \{(1, 2), (2, 1), (3, 3)\}$, $h: \{(1, 1), (2, 2), (3, 1)\}$ and
 $s: \{(1, 1), (2, 2), (3, 3)\}$ find $f \circ g, g \circ f, f \circ h \circ g, g \circ s, s \circ s, f \circ s$.
31. Let $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$ then prove that $f \circ (g \circ h) = (f \circ g) \circ h$.
32. Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) \mid x - y \text{ is divisible by } 3\}$. Show that R is an equivalence relation.
33. If $f: R \rightarrow R$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.
34. Let $A = \{0, 1, 2, 3\}$. Construct relations on A of the following types:
- not reflexive, not symmetric, not transitive.
 - not reflexive, not symmetric, transitive.
 - not reflexive, symmetric, not transitive.
 - not reflexive, symmetric, transitive.
35. Let $A = \{0, 1, 2, 3\}$. Construct relations on A of the following types:
- reflexive, not symmetric, not transitive.
 - reflexive, not symmetric, transitive.

- (iii) reflexive, symmetric, not transitive.
- (iv) reflexive, symmetric, transitive.